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THE MATHEMATICS TEACHER

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CONTENTS

Photograph of the New President 193
The Case for Mathematics . . . . . William E. Wickenden 195
Some Proposals Regarding the Preparation for Teaching High School Mathematics . . . . . E. R. Breslich 200
Adjusting the Program in Mathematics to the Needs of Pupils . . . . . G. E. Hawkins 206
Adjusting the Teaching of Mathematics to the Requirements of General Education . . . . . Walter H. Carnahan 211
The Junior High School Textbook—Plus . . . . . Veryl Schult 217
The Case of Decimals vs. Common Fractions . . . . J. T. Johnson 221
The Review versus the Telescoped Reteaching of the Work of Preceding Grades . . . . . R. L. Morton 225
Mathematics for the Non-Academic Student . . Anthony Marino 229
The Art of Teaching Methods Used in the Selection of Pupils for the Study of Algebra and Geometry in Cleveland . . . . . Ona Kraft 236
Record of Attendance, Twenty-Fourth Annual Meeting, Cleveland, Ohio, February 22-23, 1946 240
Institute for Teachers of Mathematics (Arithmetic into Calculus) Duke University, August 8-17 243
Editorials 245
In Other Periodicals . . . . . Nathan Lazar 246
News Notes 247

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THE MATHEMATICS TEACHER

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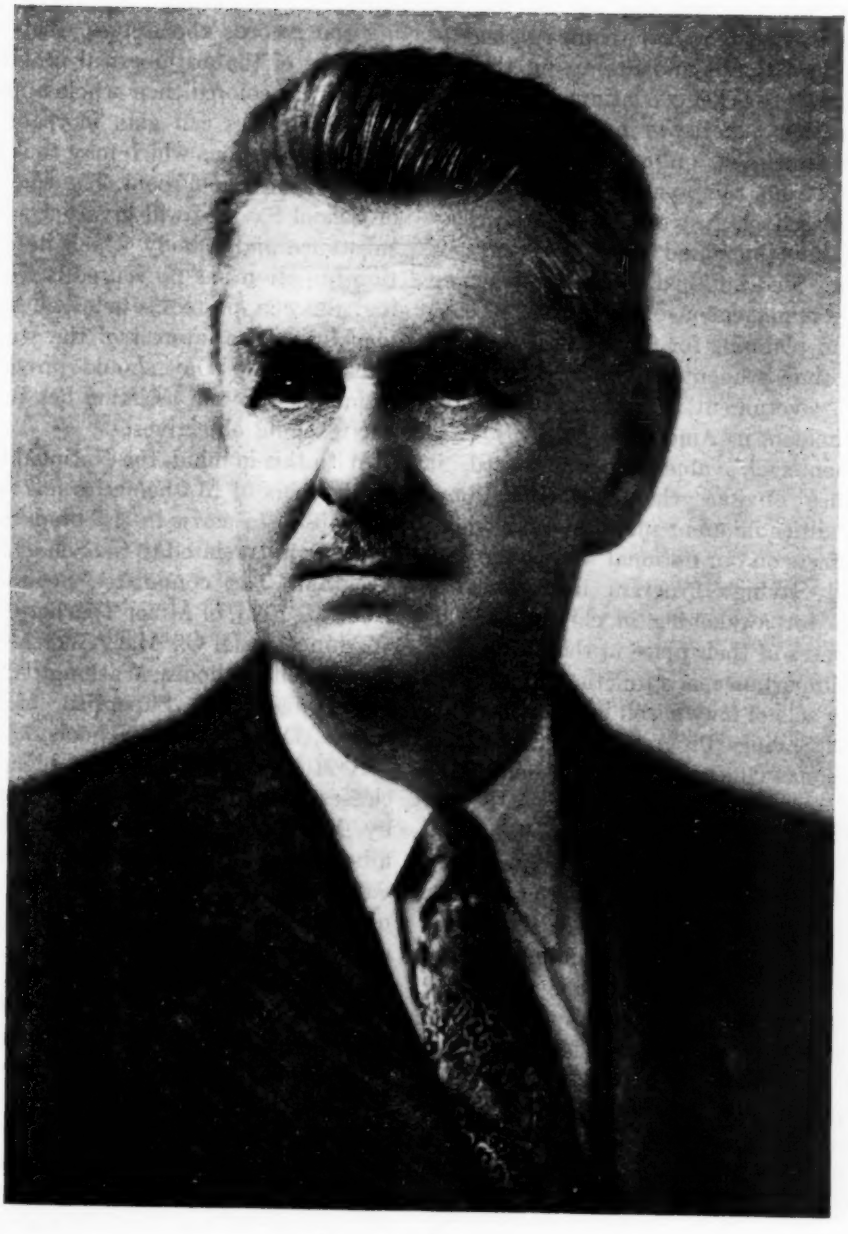

CARL N. SHUSTER

The New President of the National Council of Teachers of Mathematics

The Mathematics Teacher and the School Savings Program*

On the recommendation of leading educators throughout the country, the sale of Savings Bonds and Stamps in the nation's schools is being continued.

Wartime experience has high lighted many direct and indirect advantages resulting from the weekly opportunity to save at school. Not only has Stamp Day resulted in increased student savings, it has provided an ideal means for teaching money management—the wisdom of saving and planning for future needs. At the same time students, by participating in their government, may become active shareholders in America.

When such values are considered, it becomes obvious that the educational possibilities inherent may have far reaching effects on our national well-being. The School Savings Program has proved a means for awakening in children a consciousness of their place in the nation and their importance as a functioning unit.

Any school may enroll in the peacetime School Savings Program. The State Savings Bonds Offices will send free of charge to a principal requesting it, a School Savings Charter to be displayed in the school as evidence of active participation in the program. The text of the Charter outlines the general objectives as follows: "... to promote understanding of the national and personal reasons for saving and to give students the opportunity to save regularly at school for the purchase of U. S. Savings Stamps and Bonds."

The School Savings Program is particularly educational when it becomes an integral part of the school system instead of an extra-curricular activity. Wherever students direct Stamp Day activities the results are outstanding—not only in the dollar volume of sales but in the deeper insight into curricular studies. When children can learn to handle money by making change for their fellow students, by balancing the records themselves, their understanding of the arithmetical problems involved has colored their whole approach.

Mathematics can gain in reality from the real situations which may thus be provided in the school room. The whole scope of School Savings will give her subject a substance and a body which its abstraction too often has prevented it from having. Since a prerequisite of all effective teaching is the interest of the student, a weekly Stamp Day should prove of inestimable value in assisting her to realize her teaching objectives.

With this in mind, the National Council of Teachers of Mathematics has set up a committee to assist in the preparation of teaching aids related to the School Savings Program. The committee members are Irene M. Reid, of Miner Teachers College, Washington, D. C., Mary Worthley, District Public Schools, Washington, D. C., and Louis Thiele, Supervisor of Mathematics, Detroit Public Schools.

A study unit for junior high school classes in mathematics has been prepared by Mrs. Reid and is now available in mimeographed form on request to State Savings Bonds Offices. Similar studies are now being prepared for senior high school classes and elementary school classes. Also available for teachers of mathematics is a study unit, "Lessons in Budgeting," containing discussion questions and projects on planning and saving.

These classroom aids provide direct correlation between student savings practice and arithmetic objectives. Leading teachers and special school systems which prepare similar teaching aids can materially strengthen the nationwide School Savings Program by passing on their study guides and lesson plans. Such material will be welcomed by the National Council committee or by the Education Section of the U. S. Savings Bonds Division, Treasury Department, in Washington.

* Prepared by Education Section, U. S. Savings Bonds Division, Treasury Dept., Washington, D. C. April 5, 1946.

THE MATHEMATICS TEACHER

Volume XXXIX Number 5

Edited by William David Reeve

The Case for Mathematics*

By WILLIAM E. WICKENDEN—*President*
Case School of Applied Science and the American Institute of Electrical Engineers

THE long tide which swept our young people into high school and college over the last seventy-five years appears now to be mounting to its final surge. We are told that during these years our population increased three-fold and our high school enrollments ninety-fold, so that all but one-sixth of our young people reach high school and one-half complete its program. At the pre-war level, about one-sixth of all our young people entered college and about one-third of this number continued to their degrees. Inevitably, secondary education has been largely re-organized around social planning and personal adjustment as its coordinate foci, while the axis of higher education has shifted from the liberal to the functional area. How inevitable and how serviceable these changes have been, we who are close to the scene may find it hard to realize. A shrewd and sympathetic alien observer is likely to have them in truer perspective. For an appraisal of American secondary schools let me suggest a little book, *The American Character* written by a Cambridge University professor D. W. Brogan. My school-men friends call it the best thing of its kind and I am ready to agree.

* Address delivered at the Annual Banquet of the National Council of Teachers of Mathematics.

For all but a few of our young people, the expectation of any great economic advantage from education is receding. Prior to the war only one-sixth of our families had an annual income above $2400; this ratio, you will note, just matches the pre-war ratio of college admission among our youth. Now, the differentials between all white collar occupations and even routine skills are disappearing before our eyes. (Last year the average factory employe's wage in the Westinghouse Electrical Corporation was $3000.) The C.I.O. is candidly seeking to force the pace of technical progress in industry and to make good its claim to all the fruits in advance. A 30% wage rise based on one-sixth reduction in working hours is in effect a 10-year pay-off in advance. Caught in a world-wide levelling movement, education is losing its pecuniary advantages. The heritage of our intellectual life and culture, as a goal of education, begins to stand forth again on its own, less confused by materialistic motives.

Against this back-drop, let us re-examine the outlook for mathematics. Ground has been lost, of course, during the long-swing personal adjustment phase of the educational tide. Ground was lost with the abandonment of the doctrines of discipline, as we ceased to view the

mind as an organ, much like a muscle to be developed, and came to view it as a bundle of nerve junctions and reaction arcs, of adaptations and of skills, each to be built up independently.

Even though we hold all education to be functional, there is still a universal case for mathematics. This may be plain or fancy heresy, but I am old-fashioned enough to believe in the moral values of discipline. Perhaps I could not go so far as my old friend George Barton Cutten of Colgate, who used to say "I don't care what young people study in college so long as they don't enjoy it." I assume he was not being literal. As a moralist he was just underscoring the idea that what gets one over the tough places of life is not any particular bit of knowledge or trick of skill, but the habit of doing the thing that has to be done, when it ought to be done, whether one wants to do it or not. One might study mathematics for worse reasons.

I still believe that there are values, rewards and sorts of development to be had from swinging a pack onto one's back and spending a couple of weeks covering steep and rough mountain trails, that cannot be got from a two-hour trip through the same region over concrete pavements in a sight-seeing bus. I did the White Mountains the hard way and the Dolomites the easy way, and I retain a far more vivid and intimate impression of our New England highlands. This experience makes me a little suspicious of the more sweeping claims of progressive education, which at its best may be an ideal discipline for youthful Mozarts, but at its worst a little more than a sight-seeing tour. What if it does take ten years to climb to the snow belt on the rugged trails of mathematics and what if there is heavy work on ice slopes and rock-walls beyond? There are still enthusiasts passing through the Alpine village bearing this strange device "Excelsior." May their tribe increase!

We cannot be surprised in an era of hazy and romantic ideas on what is true, good or right to find most men shunning the rigorous discipline of deductive mathematics. A generation given to wishful thinking has few Lincolns studying geometry to learn the meaning of the verb *prove*. If mathematics is to be limited to the purely functional, we are likely to throw out the baby of logic with the bath of discipline. For most students school offers no other experience with rigorous thought processes in a zone where right and wrong are certainties and no twilight zone of plausibility exists, than mathematics affords. What we call truth, of course, is not all of that sort. Few persons realize how much that we consider knowledge or truth lies not in the domain of certainty but of probability, how much of it is relative not absolute, how much empirical and not metaphysical. If few men have any other experience of rigorous logic than the discipline of mathematics and science, then it is equally true that few will gain any grasp of empiric and inductive logic by any other approach than elements of statistical method. This has its place, I am convinced, even in elementary and secondary education. It is one of the universals of mathematical experience, on a par with fractions. All men have to deal with biological, human and social values. How far to trust and when to doubt the witness of experience or the dicta of supposed authority must be forever in doubt to a person without some foundation in the elementary logic of statistics.

Another universal, which is an indispensable tool of all modern thinking, is the functional variable. To be intensely contemporary, take this chain—wages control costs, costs control prices, prices control living standards, but none in direct ratio. Wherever simultaneous variables are interrelated visual aids to understanding are fairly indispensable. In many situations trends are more important than amounts. I have even heard a theologian define salvation as a derivative with a plus sign rather than any definite level of virtue or grace. We have grown heavily

dependent on visualization at every level of quantitative thinking. We make the mistake of giving people figures when we ought to give them pictures. This, I believe, is why most people find financial statements so mysterious and why so many have purely romantic notions on economic matters.

It is related that when Harry Hopkins moved over from the Relief Administration to become Secretary of Commerce his bureau chiefs came in one by one to meet the new boss. In his turn in came the head of the Statistical Division. "Mr. Secretary," he said by way of introduction, "You may be interested in some of the work we are doing. We are trying to find out what happens to the income money that various sorts of business and industrial enterprises have left after they have paid their taxes and insurance and bought their materials, fuel and supplies. Here are some charts covering the manufacturing industries; as you will note, eighty-six cents out of every dollar remaining goes out in wages and salaries, leaving fourteen cents for contingencies, some return on the investment and the upbuilding of the business." "Well," Mr. Hopkins replied, "I have never looked at things just that way. My viewpoint has always been the welfare angle of the social worker. But if your charts are right—and I have no doubt they are—haven't our labor friends got to get a new slant on the wage issue? How can they hope to get much more by taking it away from some one else? If they want more, they'll just have to produce more." Would that every New Dealer could get economic religion so readily! I have a friend in industry that has won three labor elections against the C.I.O. almost single handed. He hires the Cleveland Public Auditorium, then invites all his workers in for a big show. The climax comes when he dramatizes his company's operating statement. "Boys, you might like to know where our money comes from." Down the aisle comes a parade of huge many-colored boxes, each representing one source of income, to be piled up into a great cube." "Now you might like to know where it goes." Down comes another parade of boxes, including a great thick one for wages, a fairly thin one for salaries and finally a very thin one for net earnings to pay interest and modest dividends, cover risks and create new jobs by expanding the business. If the workers at General Motors knew the facts of life as well, we would never had had this fearful bloodletting just when we need all our strength to produce.

Mathematics is not just a formal, abstract language, designed to put truth into symbolic capsules. The world is literally alive with it. My fellow Clevelanders will remember Henry Turney Bailey, a vivid personality who used to direct the local School of Art. He once lectured to our engineering students on what to look for in nature. He began by rapping with a pencil on the desk: - - - - - - - - "*monotony*"; — — — — — — — — "variety"; — — — — — — — — — — — — "rhythm." Every man was alert. Then he picked up a handful of twigs he had gathered from some shrubs on the way to the hall and showed how successive branchings in length and thickness made a regular converging series. Partially breaking small branches, then peeling the unbroken bark, he produced perfect spirals. His world was alive with mathematical rhythms and no one could miss the contagion.

If we teach language only as formal grammar or philology, we make it dead. When we teach its function, we make it live. But we could also half kill it by making it a mere bag of phrasebook tricks. Give us teachers whose world is alive with beauties of form and orderly relationships, and who can make men see them in every experience instead of manipulating meaningless equations, and we will not lack students of mathematics.

The great realms of functional mathematics are the physical sciences and technologies and all manner of statistics. No man can enter these areas so vital to

modern life without his mathematical passport. The four most shocking and costly deficiencies revealed by our war experiences were physical defects, psychoneurotic instability, inability to read and mathematical illiteracy. It would be hard to say which gave our military leaders more trouble and concern. In mechanized war the mathematical illiterate is not much more than a liability. Tests given to selected high school youth for Army and Navy training programs showed startling variations in proficiency in science and mathematics over the country. There was a close correlation with per capita expenditures on secondary education and with the maintenance of intellectual goals, as against easy-going cafeteria ideas in education. The wide prevalence of such deficiencies can be a great handicap to great regions ambitious to play a larger role in the nation's industrial economy.

The world is growing steadily more technological. Not only industry, but commerce, agriculture, journalism, and homemaking are included in this trend. One can say with emphasis that the demands of this technological world cannot be met by the mathematics of the elementary school. Secondary education without mathematics leaves technology almost a closed world. It is not merely a matter of algebraic formulas or geometrical forms or statistical relations; it is more a matter of characteristic modes of perception and thinking. One can safely counsel a boy to *stay out* of the technological world if it is not second nature to him to think in dimensions, magnitudes, ratios, percentages, roots, powers and functional dependence, to *stay out* if his sense of form is weak and his powers of visualization are low.

The mathematician is rapidly coming into his own in the technological world. World War I called the chemist out of the academic shade and made him a technologist; World War II has done the same in turn for the physicist; and if there ever is a World War III it will be the mathematicians' turn in an equally big way. Peace may do the same with less drama. The search for security and prospecting for new wealth is going to be done hereafter in the laboratories of science and the mathematician is the guide who leads the experimenter to pay dirt in hitherto unexplored regions. He proved he could do it, at immense saving of time and effort, in our war research agencies.

My expectation is that this emergence of the mathematician as a practitioner will mean much to the entire profession. It has been my observation that teachers command much more money and esteem when they have counterparts in the realm of active practice and when doors swing wide between the two realms. This is certainly true of engineers and the esteem of the practitioners for the teachers is one of the strongest bonds holding able men to teaching chairs. It is probably equally true of lawyers and physicians, of architects and chemists, and seems to be becoming true of physicists, psychologists and mathematicians. Out of a pre-war staff of eleven mathematicians on our faculty at Case six have given full time off-campus on technical research, one part-time off campus and one part-time on campus. It has been a highly invigorating experience. Incidentally, we have had to raise their salaries to get them back. All teachers of mathematics will profit as this practice grows general. It is the definitive answer to the old canard "Those who can, do; those who can't, teach."

Your invitation has been an attractive opportunity to offer encouragement and express gratitude. We in technological education are deeply in your debt. What you do serves us as an invaluable screening of abilities; it is an indispensable training in laws, processes and logical habits; it is a much-needed indoctrination in ideas and ideals of mental integrity, as necessary in science as cleanliness is in medicine; it is a spring of enthusiasm to youth of creative and analytical bent. In hours of discouragement, and

there must be many of them in a world dominated by ultra-educationists, it may help to remember the Old Testament story of Elijah, who had been having a tough time with Jezebel and was feeling mighty down. The Lord found him in a cave, and asked "What doest thou here, Elijah?" And he said, "I have been very jealous for the Lord God of Hosts; for the children of Israel have forsaken thy covenant, thrown down thy altars and slain thy prophets with a sword, and I, even I only, am left and they seek my life to take it away."

Then the Lord called him forth and gave him a vision. There was a great and strong wind, but the Lord was not in the wind; then an earthquake, then a fire, but still no Lord; then a still small voice telling him a new day had come and commanding him to anoint a new king and a new prophet; then finally these words of assurance, "Yet have I left me seven thousand in Israel, all the knees which have not bowed unto Baal and every mouth which hath not kissed him."

In the name of the seven thousand teachers of engineering who have not bowed to Baal, I salute you.

GENERAL MATHEMATICS

Workbooks 1, 2, and 3 for Grades 7, 8, and 9

By WILLIAM DAVID REEVE

Teachers College, Columbia University

In these three workbooks for the junior high school the placement of each division of mathematics and the emphasis upon each follow closely the recommendations of the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics in the Fifteenth Yearbook—*The Place of Mathematics in Secondary Education.*

Covering arithmetic, geometry, algebra, and trigonometry, the drills are so devised that they may be used either for teaching or for testing. They save the teacher hours of time that otherwise have to be spent in organizing proper drill material. There is a comprehensive variety in the types of exercises, and the material is ideal for emergency courses for high school pupils generally who are going into war service.

The author has had long experience as a high-school teacher, has trained hundreds of mathematics teachers in his graduate classes, and has written many successful texts.

The Odyssey Press **386 Fourth Avenue, New York, N.Y.**

Please mention the MATHEMATICS TEACHER when answering advertisements

Some Proposals Regarding the Preparation for Teaching High School Mathematics

By E. R. Breslich
The University of Chicago, Chicago, Ill.

Those teachers of mathematics whose teaching experiences extend over a period of forty years, or more, can recall how easy it was in the early years of the new century to qualify as a teacher of high school mathematics. A college graduate with a bacherlor's degree, with a sequence in college mathematics, and with a minor in one related subject, such as physics, chemistry, or astronomy, was rated as "well prepared." If he obtained the recommendation of the department he had no difficulty in securing a position in a good high school. If he continued his studies while in service and acquired a master's degree he was looked upon as unusually well prepared.

Requirements in professional preparation were practically non-existent. Some universities were beginning to offer courses in the "pedagogy" of mathematics, but such courses were not really considered important, since it was generally believed that good teachers are "born" and that little can be done to improve the others.

As in the years that followed the requirements for teachings were gradually being raised, new courses in education were developed. They soon formed not only an important, but also a necessary part of the teacher's preparation. For education was, and is, constantly undergoing changes. Indeed, when the program of preparation for teaching a subject becomes so rigid that nothing more can be added, the position of that subject on the curriculum will soon be endangered.

We are now living in a time when more than ever before is being done to prepare the future teacher of mathematics for his work. It really is becoming difficult to see how further additions to the training program can be made. Today the prospective teacher of mathematics is expected to be thoroughly grounded in his subject. He also must be able to qualify at least in one other subject, such as physics, chemistry, general science, surveying and astronomy, if he is to command and retain the respect and confidence of his colleagues and pupils. He must be a student of the history of mathematics to understand the part mathematics has played in the development of civilization, and the place it has today in the life of the people.

In addition to this knowledge of mathematics and related subjects he must master the most effective teaching procedures; he must understand the psychology of the teaching process, and of the learning process of the pupil; he must be acquainted with the history of education; and he must be able to use and interpret educational tests and measurement. He is required to take courses in methods of teaching mathematics, in observation and in student teaching, to be prepared to deal effectively with the problems which he will encounter in his own classroom.

Furthermore, he must become a reader of the mathematical literature. For he will receive a great deal of help in his teaching if he acquires the habit of reading regularly the reports on ways of improving the teaching of mathematics which are published in the mathematical journals and yearbooks.

It seems logical to assume that during the period of years in which such an intensive training program was being developed there had been a steady improvement in the teaching of mathematics; and that this in turn would be reflected in a growing interest in the subject on the part of the pupils and administrative officers of the schools. This should produce a parallel

growth in the popularity of mathematics. The surprising fact, however, is that this has not been the case. Instead, the popularity of mathematics has been on the decrease. While at the turn of the century approximately 57 per cent of the high school pupils were taking algebra and 30 per cent geometry, the record of 1935 shows that these per cents had dropped to 25 per cent and 15 per cent, respectively. The time had come where mathematics no longer occupied the prominent place on the curriculum which it formerly held.

In trying to explain the causes of this loss of popularity, many writers have blamed it on "inadequate teaching," others on the persistent denials of the values of mathematics. The fact is that there are many causes that have contributed, not the least of which being the keen competition with the newer subjects demanding recognition. It is unfair to place the blame entirely on poor work done by the teachers of mathematics. Perhaps the present generation of teachers has been so confident of the great values which the pupils may derive from the study of mathematics that they did not take seriously the danger to it. Perhaps they did not even consider it worth while to come to its defense when they became aware of the danger. Apparently they have not been as aggressive in laboring for their cause as the teachers working in the other fields.

In contrast to the confidence which the teachers of mathematics have in the values of their subject are the activities of those who bluntly deny them; not only the educational and cultural values but also the practical and vocational values. Indeed, some question even the values of mathematics in the affairs of every day life. Often their attitude has been definitely hostile. Schools have been admonished to "prevent" pupils from taking mathematics, and pupils have been assured by educational advisers that it is a "worthless" subject. When such opinions are permitted to stand without challenge it is not surprising that pupils choose another subject in place of mathematics.

This places a serious responsibility on those who are charged with the preparation of prospective teachers. They must teach their students the ways and means which will save them from repeating the mistakes of the past which have permitted mathematics to be crowded out by other subjects, that are not nearly as valuable to the pupil. What the position of mathematics will be ten or fifteen years from now depends largely on the type of training offered to men and women who are now preparing to teach. When they go out to teach they must be well fortified with convincing arguments which they may use whenever their subject is unjustly criticized or attacked. They will have to "sell" mathematics to pupils, parents, and school administrators.

Usually the man or woman planning to become a teacher of mathematics makes his choice of subject because he is interested in it. Also it is only natural that he is thinking of his own comfort and pleasure. It is entirely justifiable that he decides on teaching as his future occupation because he believes that it is a pleasant way of making a living. Thus, he comes to the training school motivated by two strong incentives: interest in the subject, and the desire to make a good living by teaching it. But as powerful as these two incentives are, they are not enough. His enthusiasm must be aroused. While he believes in the values of mathematics, his pupils, their parents, and his principal may remain indifferent, even hostile. Enthusiasm does not develop automatically from the study of a subject. A teacher may surpass all others in the school in achievements but may fail to inspire others. Hence, one objective of his training should be to develop an enthusiasm which will last long after the training period has ended, and which will be a driving force to pass on to others his belief in the importance of mathematics.

However, not even knowledge of subject, interest, and enthusiasm, as valuable as they may be, are fully sufficient. A still higher incentive must take hold of his mind: the "desire to serve others" by making available to the largest possible number the great advantages which they may derive from the study of mathematics.

Hence, a major objective of the preparation for teaching mathematics is to make the student thoroughly familiar with the values which he may justly claim for his subject, especially the educational, vocational and social values. He must be able to substantiate his claims with convincing facts and arguments. He must be taught how to aid the pupil in the acquisition of these values.

There was a time when the position of mathematics on the curriculum could be justified mainly because of the great disciplinary values which it was believed to possess. This was not an argument invented by the teachers of mathematics to strengthen and maintain that position. Rather it was the widespread opinion of educators, school administrators, and the general public. It was commonly believed that some subjects, mathematics one of them, automatically sharpened the minds of the learners.—Today, of course, nobody believes that any subject possesses such remarkable power. However, one reason why parents send their children to high school is that they believe that it is a major function of the school to improve the pupils' mental powers, such as the quality of his thinking; his power to analyze, to generalize and to sense relationships; his judgment, foresight, imagination; and his ability to express ideas clearly and precisely. To the realization of these aims the total educational program is expected to contribute; and men and women preparing to teach mathematics must be made fully aware of their opportunity and responsibility of sharing in this program.

Evidence that these objectives are clearly recognized by the leaders in mathematics is the statement of the National Committee of 1923 which lists the following educational aims: "to develop powers of understanding and of analyzing relationships of quantity and space; of insight into, and control over our environment; of an appreciation of the progress of civilization in its various aspects; and to develop those habits of thought and action which will make these powers more effective."

It may seem trite to say that the values claimed for any subject exist only so far as they are attained by the pupils. Yet, to accomplish this is a problem to which the instructors preparing men and women to teach must give the most careful attention.

Now, since mathematics is greatly concerned with teaching the processes of clear thinking and reasoning, it is a subject peculiarly well fitted to make a direct contribution to the educational aims. It continually exhibits good models of thinking. It can be taught as a way of thinking. This is especially true of logical geometry but it should and can be made characteristic of all mathematical instruction. Moreover this same ability to think clearly is essential in all life situations wherever conclusions are to be reached by the process of reasoning from given assumptions. The students should, therefore, be taught how to offer their future pupils practice in the use of these processes, not only with mathematical materials but also with a large number and variety of life situations.

However, the teacher who plans to bring life situations into his work should be made aware of the pitfalls of the method. He must know how to select those situations which his pupils understand and appreciate. Otherwise, they will be of slight value as far as training in reasoning and thinking is concerned. Also he must avoid the danger of spending too much of the time allotted to mathematics on discussions of the life situations. The purpose of life situations is to motivate the study

of mathematics, and while the pupil should become interested in them, he should not be allowed to become so absorbed in them that he loses his interest in the mathematics which he is supposed to learn. The idea of using life situations is most commendable but the extreme use of that method involves some dangers to the subject.

When the pupil reaches high school age one of his major problems is the choice of the right vocation. He must not make a mistake in the selection of the courses which give him the best possible preparation. Here he is in need of expert advice which he usually tries to obtain from parents, friends and teachers. The advice of his mathematics teacher should be especially valuable because the uses of mathematics have been on the increase in ever so many occupations in which young people engage. The prospective teacher therefore, should be made familiar with the mathematics needed in a large variety of occupations in order that his advice to the pupils may not be vague and meaningless. With this information in his possession he should be able to supplement his advice and recommendations with specific examples of the uses of mathematics in specific vocations.

Of course, those pupils who expect to take up engineiern g, the sciences, or a profession, those who prepare for research in industry or the sciences, and most of those who plan a college education are usually aware of the importance of mathematics. But the per cent of pupils belonging to that group is comparatively small. It is the larger group which usually is in need of advice. They hear rumors that people in general can "get along very nicely without mathematics"; that it is a rather "useless" study; and that, if by chance they should find later that they need mathematics in their occupations they will be taught as they "learn the job." Only too often they accept this advice because it comes from people they know and whose judgment they respect.

Now, leaders in industry are repeatedly warning that there is an increasing need for mathematics in industrial employment. Draftsmen, pattern makers, designers, machinists, tool makers, foundry workers, sheet metal workers, electricians, plumbers, and those engaged in numerous other types of work need mathematics varying from arithmetic up to, and including, high school trigonometry. Even unskilled workers need some mathematical training. The lack of it is the cause of the loss of thousands of hours of production. There is delay and annoyance whenever a worker has to consult his superior for help on a mathematical problem. To avoid this, industry more and more selects men and women for the good positions on the basis of what they know —and lack of mathematics is a stumbling block to many who otherwise would be qualified, especially if they possess real mechanical ability. Moreover, they fail to rise in position unless, and until, they make up their deficiencies. Then they find it necessary to take courses in trade schools, business schools, and correspondence schools which they could have taken easily and more profitably in the high school. It has been found that 40 per cent and more of former students have regretted not to have taken more high school mathematics. It has been their experience that persons with mathematical training have an advantage in seeking employment. They also have an advantage in their work. Surely this is a matter which deserves the serious attention of those teachers who advise pupils as to their program of studies. It is therefore of concern also to those who prepare men and women for teaching.

As important as the educational and vocational values of mathematics are, the student preparing to teach should be made to realize that the social values are no less important to his pupils. Indeed, some writers believe that they are the most important. In modern life the majority of people have experiences in which they

come in contact with mathematics. The time has passed when mathematics is needed by only a small group of experts. It is therefore surprising and hard to understand that there are still people who claim that they are "getting along very well without it." The fact is that they are "not getting along well" whenever they meet a problem which requires some knowledge of mathematics. Then they become confused and bewildered and have to appear to others for assistance. When people need to use mathematics they generally wish that they knew more mathematics.

Moreover, magazines and newspapers are filled with discussions of a mathematical nature which an intelligent reader is espected to comprehend but which he fails to appreciate unless he has had the necessary mathematical training. The homemaker, too, needs it to be an intelligent consumer. She meets problems which arise in making purchases, paying taxes, planning budgets, investments and insurance. When she shops she must be able to select the goods which give her the largest quantity for the money spent. She must understand beauty of forms and designs with which to make the home surroundings beautiful.

In fact, anyone without mathematical training cannot really be called a cultured and well-educated person. He cannot enjoy the beauty and forms of the imposing structures of his city. His mind cannot grasp the size of the atom or the distance to the nearest star. He cannot understand our own solar system, and the movements of the heavenly bodies will always remain a mystery to him.

The student preparing to teach should therefore, at some time make a thorough study of the uses of mathematics in the affairs of every day life, not only of that which is actually used but also of the mathematics which could be used advantageously and which should be used.

Some writers commenting on the decrease of the popularity of mathematics have blamed it on the assertion that the traditional courses fail to supply the best type of mathematics which people really need while often the content is carried far beyond these needs.

In recent months various groups of teachers have been making a study of the problem of developing new courses more suitable for the large number of pupils who will need mathematics in future vocations and in every day life situations. Ultimately more suitable courses will emerge. They will be elected by the pupils who are not in the traditional sequence. It is hoped that the day will come when such courses will be required for graduation from the high school. This makes the development of these courses a task of greatest importance, since they may be a step toward the solution of the problem of the mathematical education of high school pupils. The ground work for the preformance of this task should be done in the schools which prepare men and women for the teaching of mathematics. They are the people who will try out the new courses, who will improve them on the basis of teaching experience, and who will make them a success or failure.

It has been pointed out that one of the major criticisms of high school mathematics is that the teaching has been inadequate. Of course, inadequate teaching is a perfectly safe criticism to make. No one will deny that improvement is possible even when the teaching is good. No doubt an extensive list of uncomplimentary comments on the teaching of any subject could easily be made. The following, however, will be sufficient to illustrate how such criticisms may lead to improvement of teaching.

First, there is the criticism that pupils do not know how to use their mathematics when they meet problem situations where they need mathematics. This complaint comes from many groups; the teachers of other school subjects, the teachers of advanced courses in mathematics, business men, industrial concerns and recently the military authorities. Why

are they unable to use their mathematics? There are many answers to this question: Perhaps they never mastered the fundamentals. Perhaps instead of learning mathematics they merely memorized it. Perhaps their teachers encouraged mechanical performances rather than understandings. The schools in which these teachers were prepared may not have given enough attention to the teaching process, or the learning process of the pupil. These and other answers suggested by the question, may be used to stimulate class discussions which should lead to improvement of teaching.

A second rather common criticism of the teaching of mathematics is that the pupils dislike mathematics. A pupil's like or dislike of mathematics is often merely a reflection of the teacher's ability to arouse his interest. Many pupils like mathematics even if they are not successful with it. But when his failure to understand the work continues too long, the pupil will become discouraged and his interest ceases. He may even develop a strong dislike for it. The prospective teacher should be taught how to deal with this important problem. He should know how to identify, as early as possible, those pupils who give signs of showing dislike. He should know the best procedures to help them overcome it. This really is not as difficult in mathematics as in other subjects. No subject has more interesting applications and a larger variety than mathematics with which to make an appeal to the pupils' interest. Especially helpful are applications taken from other school subjects. They will serve a double purpose. They motivate an interest in mathematics and they also contribute to the understandings of mathematics.

A third criticism of the teaching of mathematics is the complaint that the teachers make the study unnecessarily difficult, so much that some people believe that a special ability is needed to study high school mathematics. This, they say, explains why such a large number of pupils fail to make passing grades. Mathematics is not an easy subject to teach as some critics would have us believe, but it can be presented in a simple way. At least the teachers should be reasonable and not demand of the pupil that which is impossible. On the other hand, it is not the best thing for a pupil if some of the school work is not of some degree of difficulty. He needs that if he is to learn to overcome obstacles and to enjoy carrying a difficult task to completion. If school work avoids everything that is difficult it can hardly be regarded as the best preparation for future citizenship.

The prospective teacher will profit from criticisms like the foregoing if he cultivates a friendly attitude toward them. If he does not welcome them, at least he should not resent them. Instead of looking for excuses for his mistakes he should search for ways and means of removing cause for them. A student so trained cannot help but improve the quality of his teaching.

A Cone I Hold

A simple cone within my hands I hold;
Its sliced up surface Archimedes knew,
But scarcely glimpsed the wonders it could do.
Along its hyperbolic curve of old
Loran's quick finger traced the pathway bold,
While Superforts to fated cities flew.
The bombs from high upon their mark fell true,
As its parabola the slope unrolled.
Elliptic patterns on its scar seamed face
Electrons used as orbits in their race.
The atom's nucleus, with force unspent
Till Hiroshima's blast, held all in place.
Can I be judged entirely innocent,
Yet claim as mine this lethal instrument?

P. H. NYGAARD

Adjusting the Program in Mathematics to the Needs of Pupils

By G. E. Hawkins
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Providing for individual differences is an ever present problem of teachers, particularly in mathematics. Pupils vary in their ability in past achievement, in interests, and in their needs. The educational literature furnishes numerous solutions. The suggestions include such plans as individualized instruction and progress as illustrated in the Dalton Plan, sectioning into X, Y, and Z groups, dividing a given section of pupils into two or three groups and working with these groups separately during the class period, giving the same basic content and instruction to a class and including optional work for the better pupils, and so on. However, no one method seems to furnish the single best answer.

In our school, at the present time, we are sectioning all pupils in grades nine through eleven in English and mathematics only, into three groups which we have labeled minimum, regular, and special. Under this plan, in all the other classes in school, the slow pupil is thrown into groups containing a representative cross section of his class, has the intellectual stimulus of the better pupils, and has the social contacts with representatives of the entire group. Thus this plan meets one of the most objectional criticisms of sectioning as applied to all subjects. On the other hand, it does enable the teachers in English and mathematics to adjust better to the needs of the pupils the work done in these two subjects so basic to all other academic work. In grades twelve to fourteen, of course, the individual differences persist. However, various other factors in the situation alter conditions in such a way that sectioning hardly seems to be as workable at this level.

After sectioning on basis of ability and differentiation in courses offered have been agreed upon, numerous practical problems in operating the plan are encountered. Beginning at grade nine, if some pupils are to study general or applied mathematics and others the traditional courses of algebra and geometry, someone has to decide which course a given pupil should take. Allowing the pupil free choice hardly seems to be the best answer. Furthermore, such decisions should be made before the classes meet for the first time. Within a single school system where adequate cumulative records are kept and passed along with the pupil, the data should be available for properly placing the pupil. In our own case several independent elementary schools furnish pupils for grade nine. Through working coooperatively with these principals and teachers we are able to get considerable data on these pupils at the end of grade eight. Some of these are scores on tests used by the elementary schools; others are scores on tests which the high school is permitted to give in the eighth grade.

The following items are used with a rather high degree of success in properly placing pupils into sections: mental test score, achievement in mathematics, reading ability, and a prognastic test score. For the latter, in grade nine an algebra aptitude test is used. All these pupils register for elementary algebra, then on the basis of the above data, certain pupils are assigned to general mathematics. Such pupils and their parents are notified by letter before school opens of this change, are given the essential information in the case, and are invited to call the school if there are any questions. It is rather en-

couraging to find that from seventy-five to ninety such letters we rarely receive more than two or three calls. In talking with pupils and parents on this matter, attention is centered on the interests and needs of the pupil and his probable success in the course. In order to permit transfers from one class to another the general mathematics classes and the algebra classes study the same topics during the first two or three weeks of school. Thus, if it seems that a mistake has been made in a given case it can be rectified.

It seems to be somewhat more difficult to predict success in plane geometry than in algebra and thus to place pupils in proper sections. We know that achievement in algebra does not necessarily correlate highly with achievement in geometry. Similarly, geometry aptitude tests alone are not accurate guides. However, when these items are used together with achievement in arithmetic, reading ability, and mental test scores a reasonably good job of sectioning can be done. Furthermore, in the few cases where a pupil has been placed in a wrong section, it is usually fairly easy to move him to the proper one during the first part of the year, except for one thing, frequently the pupil objects to changing his teacher after he gets accustomed to his program. Sectioning in intermediate algebra is somewhat simpler. Past achievement in algebra and a mental test score furnish a fairly reliable index to probable success.

A second problem, and a major one, relates to the subject matter content to be taught in the various sections. For the pupils who lack the ability, the interest, or the need for the usual courses in algebra and geometry, the common practice is to offer a course in general or applied mathematics. Considerable experimentation has been tried during the past fifteen years with the content of such a course or courses. Most of these courses fall into one of three general types: (a) a course in diluted algebra, (b) a course in arithmetic applied to social problems and situations or (c) a course containing considerable arithmetic and its application to social problems, the rudiments of algebra, and intuitive geometry.

The Second Report of the Commission on Post-War Plans in the May (1945) issue of THE MATHEMATICS TEACHER gives one authoritative answer regarding content for such courses. It advocates functional competence in mathematics for all who can achieve it. The report amplifies the meaning of functional competence through giving a series of twenty-eight questions for use as a check list. The content suggested includes arithmetic, simple algebra, and intuitive geometry. The mathematics teachers in our school have examined these twenty-eight items rather carefully and agree that in general these are desirable goals for all who can achieve them. Our course in ninth grade general mathematics in particular works toward most of these goals.

When some plan of sectioning is followed in algebra and geometry, the question of content to be studied in the various types of sections arises. It seems to the writer that if the courses, even for the minimum groups, are to be worthy of names algebra and geometry that certain basic content must be taught in all of them. To put it another way the various kinds of sections in algebra should in general study the same group of units. However, within each of the units there should be considerable adjustment made in both content and method to meet the needs of the pupils. In the minimum sections in geometry, for example, probably the major emphasis should be placed on the practical aspects of the subject, that is geometric information, problem solving and other applications, and constructions rather than emphasizing deductive reasoning and proof almost exclusively. This does not mean that proof should be omitted entirely for these pupils, but they definitely have more success with the practical elements of the course than with proof. For the superior pupils on the other

hand the content of the course can and should be enriched beyond that offered the regular group by including additional topics, more complicated applications, and as much of the abstract work as seems desirable. For example, in intermediate algebra the superior pupils may very well include in their work determinants of the second and third order, topics which may be omitted entirely by the other sections.

Assigning of grades to pupils in the various kinds of sections also calls for some consideration. For example, if pupils are assigned grades of A, B, C, D, and F in mathematics, should pupils doing the best work in the minimum sections receive the top grades A's and B's? Of course, the theory that sounds best to many would dodge this issue entirely by reporting in some manner on the individual growth and development of each pupil. That is, compare the present with the past achievement of a given pupil and not compare his achievement with that of other members of his group; require each pupil to compete with himself and not with the others in his class. However, regardless of how well the theory sounds, it has not been widely accepted in school practice. It seems that parents want the work of pupils graded in some manner similar to what they were accustomed to when they were in school. Then colleges and universities in their work in admissions require data on the academic rank of a pupil in his class; business and industry often require similar information in their application forms for jobs. Then in the actual life experiences of both the adult and the child on the job, on the playing field, and in social activities, competition with others even more than competition with one's self is anticipated and found in practice. Thus perhaps there are good reasons for the persistence of competition with others in school work and the practice of teachers in assigning grades.

In our school the teachers have agreed that the grades given a pupil in the various subjects should be based on the achievement of the pupil in the stated objectives of the course. Since the teacher or group of teachers in a given area has responsibilities in setting these objectives he has some leeway in determining what factors should be considered in deciding on the grade a given pupil should receive. However, the pupil's proficiency in the subject is the predominant idea in the minds of all the teachers in assigning grades. Following this plan logically leads to the interpretation that if a pupil is in a minimum section in plane geometry he should receive the same grade that he would receive for work of the same quality if he were in a regular or a special section. Hence in all but exceptional cases where a pupil is in a wrong type of section those in the minimum sections do receive grades on the lower half the grading scale. Similarly those in the special sections receive grades on the upper half of the scale. Of course the plan does not work without occasional difficulties, but they are minor. The plan seems to us to be a more fair practice than that of giving a weak pupil an exaggerated idea of his abilities and achievements. Most such pupils do not expect to receive high grades anyway. We have found it desirable to give some of the same achievement tests in all the sections of a given subject so that both teachers and pupils may have a better perspective of what constitutes poor, good, or excellent work. Then if a teacher can be assigned two or three different types of sections in a given subject he has a much better idea of what he may reasonably expect from the pupils than he would if he were teaching only special sections or only minimum sections.

For pupils studying general mathematics instead of algebra in grade nine, the school has the problem of deciding what to do with these pupils in succeeding years so far as mathematics is concerned. Several writers suggest a second track or program with a two-year sequence or more in general mathematics for such pupils. All teachers are keenly aware of the fact that the functional competence advocated

in the Report of the Post-War Commission is not attained by all slow-learning pupils by the end of grade nine. Some schools are able to extend their program in general or applied mathematics at least through grade ten. However, those schools are probably the exception rather than typical cases. Several factors seem to complicate matters and prevent many schools from offering more than one year of such a program. Textbooks for the second year of such a course have been very slow in appearing. Perhaps part of the reason lies in the uncertainty of the authors regarding the content for the second text. Probably more pertinent is the failure of school people and the authors to convince publishers that there is really a demand for the second book.

Within a given school there may be factors which discourage offering a second year's work in general mathematics. At the present time our pupils who study general mathematics in grade nine register for algebra in grade ten if they want more mathematics at that time. About two years ago we examined the school records to see what had happened to approximately ninety pupils who had studied general mathematics the previous year in grade nine. We found that about one-third were no longer in our school, that one-third were taking no mathematics in grade ten, and one-third were enrolled in algebra. Recently we made a similar study for last year's pupils. We found that about 20 per cent of them are no longer in our school, 30 per cent are taking no mathematics, and 50 per cent are studying algebra. Of those studying algebra 7 per cent are doing better than average work, 36 per cent are doing average work, 22 per cent are doing fair work, 27 per cent are just passing, and 10 per cent are failing. These latter figures furnish some evidence that mental maturity plus the training received in the ninth grade course in general mathematics may enable some pupils who in the past have been quite weak in mathematics to study algebra successfully. Many of the parents of these pupils and the pupils themselves would insist on studying algebra, at least in grade ten, if an attempt were made to put them into some other mathematics course. Some of them have plans to get as much as three years of algebra and geometry while in high school. Hence, we have few pupils left from the ninth year group who would be potential candidates for a second year of general mathematics.

At least for those pupils who do not take the courses in algebra and geometry a course in consumer arithmetic in the senior year would seem to fill a genuine need. However such a course is usually elective and frequently is not chosen by the very pupils who need it most. The lack of good texts in this area has been a handicap. Furthermore to be most effective, the work in such a course must be directed by an alert, versatile teacher who supplements the textbook with fresh problem materials chosen from current topics. Perhaps in the future such a course will have a more important place in the schedule and serve in developing a degree of competence in mathematics for a certain segment of our pupils.

The teacher of a group of slow-learning pupils soon notes several general characteristics of a majority of such pupils that makes the problem of working with them somewhat different from that of teaching the average class. As noted above there is likely to be a large turnover in such a group due to the family moving from one locality to another. Absences in such classes are greater than in other classes. In some cases these factors rather than lack of ability account for the weaknesses of the pupil. Many of these pupils dislike school work, are restless, have difficulty in settling down to work or in sticking with a job for more than a few minutes at a time. They show little independence in attacking problems and succeed best with memory-type work. Many of these pupils learn to do well jobs that they can imitate; that is, they can do problems just like the ones the

teacher explained, but in problem solving and in application of principles to new situations they are likely to rate as very poor in spite of all that the teacher can do for them. Usually they do just as poor work in other academic courses as in mathematics. Such a class presents a real challenge to the best of teachers. It can hardly be cared for adequately by giving it to any teacher who doesn't have a full program of other classes.

There is always a danger that a certain stigma will be attached to the slow sections, that they will be labeled dumb-bell classes. This situation can be averted to considerable extent by proper planning and handling on the part of the teachers. Except in cases in which pupils are taking a different course than other pupils, for example general mathematics instead of algebra, it is not necessary to announce to the group that this is any particular kind of section. While it is true that pupils soon discover that all the others in the class are "all sharks" or are "all dumb" as they say, yet the pupils make no particular point of it. In talking to pupils it is helpful to emphasize their own needs and the importance of their success in the course. While there are numerous conflicting factors to deal with in suggesting that a pupil move from one section to another, for example he may want to stay with certain friends or he may want to keep a certain teacher, yet most pupils are sufficiently concerned about doing as good a job as possible that they are willing to take advice that appears helpful.

The slow-learning pupil especially needs a patient, sympathetic, understanding teacher, one who is willing to explain a process again and again if necessary until the pupil gets the idea. We have had some interesting comments from pupils in this connection. Shortly after school opened we suggested to a few pupils in general mathematics that they try the algebra course instead. Our problem was one of providing space for some new entries who needed general mathematics worse than the other pupils and we had evidence that the latter pupils probably could succeed in algebra. Almost without exception these pupils objected strenuously to being moved to algebra. The reasons they gave were convincing. They wanted this course and this teacher, because for the first time in their school experience they could understand their work in mathematics. Success for the pupil means success for the course. We, as teachers, are well repaid for any adjustments which we can make to enable the pupil to understand his work and thus to do it profitably and perhaps happily.

HOUSEWIVES are asked to make three loaves of wheat bread do the work of five, and that their purchases of fats should be reduced 20 per cent. Public eating places have been asked to limit their purchases of wheat flour or its equivalent to two pounds a week for the average number of their customers and to reduce their consumption of fats also by 20 per cent. That is not starvation for the American people, because these amounts can be made up by other abundant foods which we have and by the saving of waste. This amount of denial alone would save the health and lives of at least 35,000,000 additional women and children. . . .

This whole problem is an issue of religious faith and morals which affects our country as a whole and every individual. Saving of human life is a moral and spiritual duty. If your neighbors and their children were hungry you would instantly invite them to a seat at your table. These starving women and children are truly in foreign countries, yet they are hungry human beings and they also are your neighbors. Could you not imagine one of these helpless women and children as an invisible guest at your table? And by following the voluntary rules for saving food you give life to that starving person just as surely as if he sat at your own table.

I can only appeal to your pity and to your mercy and your sense of service. I know the heart of the American people will respond with kindliness and be generous to all the suffering. Will you not take to your table an invisible guest?—HERBERT HOOVER.

Adjusting the Teaching of Mathematics to the Requirements of General Education

By WALTER H. CARNAHAN
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THE preacher begins his discourse by quoting a text and by telling chapter and verse where it can be found. It is a good practice which I borrow for present purposes. My text is taken from the first paragraph beginning on page one hundred-fifty-six, Chapter IV of the book sometimes called *The Harvard Report*, but more accurately called *General Education In A Free Society*.

> But as one leaves direct experience, and the immediate and familiar, an increasing need arises for an intellectual structure. The facts—and experiences—can no longer stand by themselves, since they no longer represent simple, spontaneous, and practical elements directly related to the life of the student. As they become further removed from experience, more subtle, more abstract, the facts—must be learned in another context, cultural, historical, and philosophical. Only such broader perspectives can give point and lasting value.

I have quoted a remarkable passage from a remarkable book. While the passage was written as a comment on secondary school science, it might equally appropriately have been written as a part of the discussion of the teaching of mathematics. It is the purpose of this paper to attempt an interpretation of this passage from the point of view of a teacher of mathematics and to make certain recommendations for the implementation of the principle it lays down.

Other than the expression *Theory of Relativity*, it would be difficult to find a word that has a more exclusive and cloister-like sound than *philosophy*. It would be interesting for each of us here to try out the word with some of our acquaintances in the student body of the home high school and report back their reactions. I think the list of their comments would include the following: "I don't catch on." "Am I supposed to understand what that means?", "Say that in English," "I don't speak Choctaw," "Come again?," "Phil who?" Nevertheless these same high school students are incurable philosophers. They are forever asking "Why?," "How does that come to be?," "Wouldn't some other way do as well?," "What good does it do to know this?" They question adults; they question each other; they question teachers; they question preachers; and most of all, they question themselves. And what is philosophy if it is not raising questions and trying to find answers regarding the meanings of things?

One difference between the savage and the civilized man is that the former thinks caprice controls the events of life while the latter knows he lives in a world of law and logic. By far the greater part and certainly the most important part of the process of education is devoted to civilizing young men and women by showing them the law and logic of their life experiences, physical, mental and spiritual. It is by this process that they are emancipated from the tyranny of fatalism and fear. "You shall know the truth and the truth shall make you free."

How is it, then, that we so easily fall into the habit of substituting rule, mechanism and manipulation for law and philosophy? Teachers, text book writers, and publishers must share the responsibility for this tendency about equally, I think. An author writes in his manuscript, "Mathematicians have agreed that the product of two negative numbers is a positive number," and gives no reason to justify the agreement. The editor okehs this and it goes into the book. The classroom teacher has the pupil learn the rule and use it without any attempt at rationalization.

Thus the three adults conspire to offend the intelligence of the young philosopher who wants and is entitled to have some explanation. Or again, an author writes, the editor approves, and the teacher quotes, "Division by zero is not permitted in mathematics." The young philosopher wants to know, "Why isn't it permitted?", and it is a part of the struggle from savagery to civilization that he should have an answer, patiently given and with all the background information the teacher can bring to bear. In this connection, the young philosopher may make real trouble by asking what is the result of dividing zero by zero. (The speaker once had a student who made himself unforgetable by asking this question on the average of once every two weeks.) Even here a reasonable answer is called for.

It would be interesting to know how many students (or how many teachers for that matter, present company excepted), have ever heard a logical explanation of why we invert a divisor fraction and change from division to multiplication. The fact that students do not demand an explanation is no justification for not giving it; it is the responsibility of the teacher to make this contribution to the process of civilizing them.

This responsibility of the teacher to contribute to understanding even when students do not demand it can further be illustrated in connection with the teaching of geometry. Not once in ten years will any student question the necessity of the parallel postulate. Shall we for that reason ask or permit its acceptance without pointing out alternate postulates and some of their consequences and applications? It seems to me that here is an opportunity, not to be overlooked, to contribute to the intellectual emancipation of students by at least calling attention to the existence of the roads that lead to relativity, the equivalence of mass and energy, and the atomic age, subjects intimately related to the parallel postulate strange as it may seem.

Not long ago the speaker visited a high school algebra class that had been taught to transpose, and a class discussion revealed that no one knew the justification for the operation. Even the teacher was not conscious of the fact that transposing is an abbreviated application of the addition axiom. Here again an opportunity for philosophy had been passed by.

This list of illustrations could be extended indefinitely, but enough have been given to reveal what, in the speaker's opinion, is meant by philosophy in teaching. It is not something new; it has been a part of good teaching and learning since Socrates and Euclid. It calls for wide reading, deep thinking, and skillful, purposeful, and earnest teaching. There was a time (and I am glad I can remember that far back) when philosophy was much more a part of teaching than it is today. In that time, conditions favored this type of teaching. Only one person out of every nine or ten eligible ones entered high school and that one was by nature or environment predisposed to philosophy. Now when every Tom, Dick, and Harry go to high school (and that is as it should be) sustained interest in development and discussion is impossible for many. Does this statement sound like a retraction of my former statement that students are by nature young philosophers? I think there is no inconsistency; Tom, Dick, and Harry like to ask questions and get answers but then, they also like to tickle Mary's neck, or surreptitiously read comic books. The interest span is short, and lengthy, abstract discussions go over their heads. Therefore, philosophy has to be administered in broken doses with concrete illustrations and interspersed activity periods. Discuss awhile, illustrate, take pencils and paper and work awhile, then back to rationalization.

A corollary to the great increase in student enrollment in secondary schools is the great increase in the number of teachers. And here is another reason for decrease in attention to philosophy.

Thousands of persons are turning to teaching who fifty years ago would never have considered the profession and would not have been employed if they had done so. These teachers may be better than the older teachers, or they may be poorer; the obvious fact is that they are different. They think differently and they teach differently. They *must* teach differently because they have a different type of students and many, many more of them. And so we have intelligence testing, ability grouping, multiple track courses, objective tests, standardized tests, supervised study, work books, and other such mechanics, but much less attention to philosophy. Let me repeat that I am not objecting to this new mechanization, but I believe it should be balanced by renewed attention to rationalization.

Philosophy takes time. Where is this time to be found? Is it too much to recommend that it be found by reducing the number of exercises and problems to be solved? I know a teacher who boasts that her students do thousands of exercises in geometry every year. My sympathy goes out to them. What limited understanding they must have of the real meanings involved or to what end all of this is leading or can lead. The decision regarding the redistribution of time should be based on the relative values involved, whether it is better to have much manipulation and less understanding or more understanding and less mechanical manipulation. To the speaker, it seems that better mastery demands more attention to understanding.

I have in my files a page of data given to me some years ago by an editor of a large publishing company showing the number of exercises in texts in plane geometry published by certain competing companies. The conclusion which he drew was that any new publication issued by his company must have more exercises than any competing text. If this standard of text preparation were to prevail, authorship would become a very simple matter of counting pages, words, pictures and exercises. Fortunately, authorship and publication in America are on a much higher plane than this, even for the company whose editor was so number conscious.

Perhaps there are those who believe that what is implied in more emphasis on philosophy is greater rigor in logic. Such is not the case. No one knows better than the student of mathematics the great, unavoidable limitations of logic. In a very real sense, mathematics is a game; one lays down rules and plays according to the limitations they impose. Furthermore, much of the game of mathematics is played according to conventions that may have less logic than we sometimes admit. For example: $\sqrt{a}\times\sqrt{b}=\sqrt{ab}$. But if $a=b=-1$, $\sqrt{a}\times\sqrt{b}=\sqrt{-1}\times\sqrt{-1}=(\sqrt{-1})^2=-1$ not $\sqrt{-1}\times\sqrt{-1}=\sqrt{-1\times-1}=\sqrt{+1}=+1$. Or again, $1+2\times3=7$, not 9. Convention has more to do with these operations than logic. During the Middle Ages most university students, mature men as they were, could get no further in plane geometry than the proposition that the angles opposite equal sides of an isosceles triangle are equal, the *pons asinorum*. The reason was that the subject was unjustifiably burdened with attempts at absolute logic and professors and students soon found themselves tangled like a litter of pups on leashes.

No, it is not absolute rigor that is advocated, but rather an appeal to the logical sense, which is something quite different. The minds of adolescents require a feeling that what is being taught is reasonable and justified by reasoning, not always that it is based upon uncontrovertible logic. Illustrations, growth of the new out of the old, completeness of development, revealed relationships are often quite as satisfying as formal logic.

A possible explanation, but by no means a justification, of the tendency to teach without adequate attention to meaningful development is failure on the part of some teachers, or perhaps all teachers at some times, to distinguish between what is re-

quired of good teaching and good text book writing. The writer of a text makes each fundamental explanation and carries through each fundamental development once and once only. That is all that is required of a good text. We teachers are too prone to teach on the same basis, and that is a serious error. Many students do not grasp a new principle as the result of one presentation. Explanations and developments have to be repeated with variations until the principles are thoroughly grasped by all who are capable of being educated in the subject. This repetition serves not only to clarify concepts, but gives students a sense of the importance of true understanding. The student comes to realize that if understanding is important enough to be emphasized by frequent repetition in class, it is important enough to be made a habit.

Attention is now directed to the second idea advanced in the text passage, increased emphasis on historical materials. It was J. W. L. Glaisher who said, "I am sure that no subject loses more than mathematics by any attempt to dissociate it from its history." This is as true of elementary as of advanced mathematics. Young people are intensely interested in personalities. Each is interested in the person he or she expects to join in life partnership, in the intimate friend with whom hopes and problems are shared, in actors and actresses, in those who experience the drama of great success or tragic failure. They are interested in knowing when and by whom discoveries were first made and under what circumstances, what was going on in the world when these contributions were made, how they were transmitted. The ancient Greeks had a legend that the goddess Athena sprang full grown from the head of her father Zeus. Mathematics, however, did not come into being by such a birth. It grew slowly, painfully, tragically, triumphantly, dramatically. The men who produced it were saints, gamblers, priests, liars, rulers, egotists, drunkards, public servants, libertines, blind, sick, socialites, and every one of them was a human. Euclid would be remembered to this day as a great teacher even if he had never written *The Elements*. Newton would be known as one of Britain's great public servants even if he had not written *Principia*. Descartes would still be read as a great philosopher even if he had not written his *Geometry*. The biographies of Abel and Galois would be great drama even if these men had not in their extremely brief lives developed mathematics so as to enrich science for hundreds of years.

It is not only the drama and human interest in the history of mathematics that makes it worth reading and telling. Even more than these is the light shed on the meanings of the subject. Not long ago I was visiting a class in first year high school algebra. The teacher was reviewing arithmetic fractions as a basis for work with algebraic fractions. At his request, students took pencil and paper and divided 1/2 by 1/3. William struggled with this a few minutes and sat looking doubtingly at his work. The teacher stepped to his side to see what was the matter. William had written $1/2 \div 1/3 = 3/6 \div 2/6 = 3/2$. "No, William," the teacher said, "that is wrong. Try again. Invert the divisor." William inverted the divisor, multiplied and obtained the same result as before. Then the young philosopher wanted to know "Why?" The teacher said, "Well, I don't know why it gets the same result, but that isn't the way you learned to do it, and changing to common denominators is longer and isn't necessary."

The teacher was right: that wasn't the way William had been taught to divide by a fraction, nor his father, nor his grandfather before him. But some of his ancestors were taught to do it just that way. From the time of the invention of printing, and for two hundred years thereafter this was one of the commonly used methods. Having forgotten the method he had been taught, William had reinvented a method that passed out of use three

hundred years ago. Although one may well question whether he had arrived at the method by any process other than a fortunate guess, nevertheless, here was an opportunity for the teacher to refer William to a good history of mathematics to read the history of methods of dividing by fractions and make a report to the class.

When we introduce students to geometry we rightly assume that they know what a straight line is and how to draw one, but let us slow down a bit here. There is some interesting history connected with making a straight line. Drawing a straight line is one of the most difficult problems in geometry, and the last of the necessary constructions to be mastered. Not until one hundred years ago had anyone ever known how to make a straight line by geometric (as distinguished from physical) means. A Frenchman by the name of Sarrus first showed how to do it, then Peaucellier found another way, and today some half dozen ways are known. This page from history has a contribution to make to the education of the student, a contribution at least as important as that of drawing a few more straight line figures on paper with the straight edge.

The construction of regular polygons in geometry furnishes an excellent opportunity for the introduction of a fascinating incident from the history of mathematics. Ancient mathematicians had been able to construct regular polygons of three, four and five sides, and other polygons derived from these, but beyond that they were unable to go, just as they could bisect but not trisect an angle. A little more than one hundred years ago, Gauss, then only seventeen years old, discovered a method for constructing a regular polygon of seventeen sides. This simple statement is sufficiently interesting to justify the time necessary to have some student review it in class, but of more value is the opportunity afforded for a brief biography of Gauss, one of the three greatest mathematicians of all time. He is one of those who have given us our mathematical inheritance, and students are better educated when they know something about how he lived and labored, and their continued interest in mathematics is much more likely because of this contribution to their education.

There is a strong temptation to go on and on reviewing interesting pages from the history of mathematics and showing how they relate to the teaching of the subject in high school and college, but enough has been said to illustrate the value of this kind of material, and that is all that can be done here.

Once more, let it be said that introduction of historical materials is not a teaching luxury, something to be indulged in when time hangs heavy on the teacher's hands. It is an integral part of complete teaching, an investment that pays good dividends, to be bought like an annuity by making regular contributions.

Increased emphasis on philosophy and history in our teaching of mathematics is a phase of the important program of presenting the subject as a complete, well rounded entity. It is as an entity that you and I and all those who see beauty in the subject look upon it. And it is as an entity that our students must see it if they are to have a deep and lasting interest in it. To be sure, there are many who will never have such an interest in the subject, no matter how it is presented, but we should present it in its most attractive aspect for the sake of those who are capable of comprehending its beauties as well as for the temporary benefit of those who are soon to cease all systematic study of mathematics. Perhaps students of this latter group will receive some permanent cultural benefit (to be discussed presently) even though most of the details which the teacher presents are soon forgotten.

Another phase of the presentation of mathematics as an entity which needs renewed emphasis is what we may call "the distant view of things to come." What we

here have in mind is not the removal of barriers which still sometimes separate the various branches of mathematics into watertight compartments for isolated teaching, but the occasional drawing of the curtains to reveal something of the nature of phases of mathematics which the student will not be expected to master for some time, perhaps years to come. You and I know that all mathematics has beauty, and power. Even a limited and distant view serves to reveal these characteristics. Such views should be frequent experiences of our students. They serve as invitations to intimate and prolonged acquaintance. To the high school student who says, "My brother at college is taking analytic geometry. What is that?" shall I reply, "Wait until you go to college and you will find out." To one who says, "My father says he studied calculus. What is that subject?" shall I say, "You will learn that when you are ready for it. Confine your attention to algebra for the present." In my opinion, such questions present fortunate opportunities for the teacher to cultivate real interest in mathematics and to foster a mental attitude favorable for future learning. Even when such leading questions are not asked, it is worthwhile for the teacher to take advantage of opportunities which pupils are not prepared to recognize for brief, nontechnical previews of mathematics which we hope capable persons will study later. For example, the study of tangents of circles gives a natural setting for brief mention of the calculus which grew out of the problem of drawing tangents to curves other than the circle. The study of quadratic equations provides opportunity for brief mention of conic sections the equations representing which are quadratics, and this in turn invites passing mention of analytical geometry.

There remains the third of the three ideas suggested for consideration by the passage quoted as a text. What shall we say about teaching mathematics for culture? Every complete modern manufacturing plant produces by-products which may equal or exceed in number and value the products in which it specializes. In mathematics we specialize in training in thinking, in imparting mastery of techniques and in appreciation of power and beauty. Culture is one of the by-products. Can it become one of the specialized products? I doubt it. It is by nature an imponderable; it is an intangible asset; it is spiritual; but it is not unreal and it is not without value. It will not be found on page one hundred twenty in the textbook, and it will not be evaluated by a question on the final examination, but it should be consciously sought and striven for.

And now abide philosophy, history and culture, these three, and the greatest of these is culture.

The Trials of Geometry

By Caroline Brownholtz

Characters: Miss Octagon, teacher of geometry, Miss Pentagon, Miss Supplement, and Miss Angle.
Scene: A geometry class room.

Miss Pentagon and Miss Supplement entered the square geometry room. They gazed doubtfully at the rectangular blackboards. Miss Octagon entered the class room. The girls begin to shake in their triangular shoes. Miss Angle was the first pupil to be called on to work on the hexagon problem. Her failure to complete it caused a converse among the pupils. The problem failed to penetrate many of their square heads. The class was bisected in their conclusions.

Miss Pentagon who sat parallel to Miss Angle was called on to give her plan. When she was called, she was attacked by an acute pain. Not wishing to come adjacent to Miss Octagon's straight edge yardstick, she made a complete circle of the class room. After two consecutive tries at the problem, Miss Octagon made a bee line towards her. Poor Miss Pentagon's hair stood perpendicular.

A ray of inspiration enlightened her. She was then able to solve the problem by her statements. She received a complement for her reasons.

Miss Pentagon, by the functions of her geometric mind and by the sine of Miss Angle's depression, came to the conclusion that higher mathematics was beyond the amplitude of her mind.

The Junior High School Textbook—Plus

By Veryl Schult
Washington, D. C.

This year, 1946, besides our celebrating 25 years of the work of the National Council of Teachers of Mathematics, we are celebrating the 200th anniversary of the birth of a great educator, the Swiss, Pestalozzi. And since he taught us much that applies to the teaching of mathematics today, I want to begin by reviewing briefly our debt to this man who might well be called the father of modern education.

The schools of his day, as those for the previous two thousand years, existed to teach the written word. Pupils memorized and teacher flogged unmercifully. No wonder Shakespeare speaks of the schoolboy "creeping like snail unwillingly to school." Pestalozzi's most fundamental principle of method was his "sense perception." This was his method of avoiding the "superficial verbosity" as he called it, of the Alexandrian type of school. In mathematics, he developed various charts and pyramids of figures to help children visualize arithmetic and to learn it faster and easier. He stressed what he called "mental arithmetic," that is, *understanding* versus merely following rules. Various teachers then carried out his ideas. Warren Colburn in 1821 published his First Lessons in Arithmetic, "on the plan of Pestalozzi" (as he said) "with some improvements." This book and other early arithmetics can be found in the stacks of the Library of Congress in Washington, and they furnish most interesting examples of the early arithmetics.

A little time spent with these books will make us appreciate how comparatively easy our job is now! Here are some of the things you would have had to teach if you used those books of a century or more ago: (quoting from some tables of contents) *alligation medial, alligation alternate, duodecimals, Custom House allowances, barter, permutations, gauging* (fancy rules for measuring the quantity of liquor in a cask), *single and double fellowship, cube root, and lumbermen's notation.*

Among the arithmetics in this splendid collection is the Introduction to Arithmetic by John Preston, 1834. He devotes several pages to what he calls "The Pestalozzi Plan," in which he recommends that instead of figures, boys use peas, beans, or kernels of Indian corn to perform various processes. "This experience," says Preston "will afford children pastime when they are out of school; it will keep their minds intent on the business they are sent to perform, and will not disappoint their guardians nor grieve their hard-labouring and frequently indigent parents."

Since Pestalozzi's ideas strongly influenced textbooks, which in turn so strongly influence teaching, I wanted to use this opportunity in this talk on "The Textbook—Plus" to pay tribute to him.

Textbooks have been improved so much over the years since the first printed arithmetic in Treviso, Italy, in 1478, by influences such as Pestalozzi, that one finds it rather thrilling to contemplate what they will be like in years to come. What a contribution was made by Comenius, for instance, when in the 17th century he ventured into adding pictures to textbooks to help make the meaning clear! In a modern book just the other day, I saw a reproduction of Comenius's blackbirds sitting on the horizontal boards of a gate, a picture which he had put in a textbook almost 300 years ago to help children in their arithmetic lessons.

Some of our modern junior high school textbooks are excellent, and pupils find the study of mathematics tremendously motivated by good textbooks. For instance:

Pictures, diagrams, and illustrations have been made much more interesting *and* functional. Some books even have

three-dimensional pictures, with a special lens furnished with which to view them.

The language has been brought to the level of the pupils.

Problems are more realistic and are about the concerns of children.

Design and craftsmanship have improved.

The psychological approach is used.

One arithmetic textbook which recently came to my attention even uses the sequential comic strip technique to develop a certain idea in arithmetic. There must be a universal appeal to this technique when everyone from the superintendent of schools down to those learning to read, rush to their morning papers to see what new adventures Superman, Dick Tracy, or Donald Duck, is having.

In 1844, the Commissioner of the U. S. Patent Office said, "The advancement of the arts from year to year taxes our credulity, and seems to presage the arrival of that period when human improvement must end." How little did he foresee oil being converted into rubber, glass into ladies' stockings, coal into butter, beans into buckles, etc.! And so our advances have been made in all educational endeavor.

But what a long time it has taken us to learn the lessons Pestalozzi and others like him have tried to teach us! For instance, following a textbook is not enough. One day, in one of our home economics classes, the teacher asked the girls to make half a recipe which involved taking $\frac{1}{2}$ of $1\frac{1}{2}$ cups of flour. Not one girl in the class was *sure* of the amount to use! After the home economics teacher complained to the mathematics teacher, the latter, as soon as the children returned to her, sent all the girls to the blackboard and instructed them to take $\frac{1}{2}$ of $1\frac{1}{2}$. Every one did it correctly.

"Your home economics teacher said you could not do that in class today," said the mathematics teacher.

"But," interrupted one little girl, "that was CUPS!"

This indicates again the need for relating the mathematics work, as far as possible, to actual experiences so that we will not have our mathematical knowledge "in a vacuum."

There is an old proverb that says that it is never boresome when a shopkeeper talks shop. It only gets boresome when a bootmaker tries to talk about chiffon. Therefore I am going to talk shop and mention just a few of the things we have been doing in our junior high schools in our efforts to do a better job and to show that we have profited somewhat by what teachers such as Pestalozzi have taught us.

We have recently been working on a curriculum revision program in mathematics. We tried to keep in mind some of the lessons that I have been mentioning. We started with our philosophy of education as evolved by all the teachers in the system, decided what that implied for us in mathematics, and then set up our objectives for mathematics which would be in harmony with our philosophy. We selected certain objectives which were important in the pupils' PERSONAL LIVING, those relating to their SOCIAL ENVIRONMENT, those relating to their NATURAL ENVIRONMENT, and those relating to mathematics in EXPRESSION AND COMMUNICATION.

We decided that in the interests of articulation, we needed a (so-called) Handbook for all teachers of mathematics on all levels from kindergarten through grade twelve, and also separate supplementary, detailed courses of study for the elementary, junior high and senior high level. In the Handbook, we have an orientation for all teachers which includes our philosophy of education, the implications for mathematics in that philosophy, our general objectives which I mentioned, and then a few other items such as a summary of the report of the Postwar Planning Commission of last May, notes on evaluation, historical notes, practices recommended in the presentation of cer-

tain processes in arithmetic, guides for drill, and recommendations about maintenance of skills which is a great responsibility for teachers on all levels. Following this part in the Handbook, the goals for each grade from kindergarten through grade 12 are briefly outlined.

The detailed course of study for junior high schools, which supplements the Handbook, contains the units of work for that level, each with goals expressed in terms of understandings, appreciations, and abilities, with suggested materials and references for each unit, and with teaching suggestions where we listed "ideas that have worked." These ideas were the results of our own teaching experiences, as well as wide reading. Also in the detailed courses are recommendations the Committee makes to all junior high school mathematics teachers, the section from the 15th yearbook concerning terms and symbols, some notes about a mathematical atmosphere in the classroom, and a bibliography.

A few of the teaching suggestions are as follows: Maybe they are (quoting Winchell) "things we never knew till now, but which you probably knew all the time."

A laboratory technique is most important. From Pestalozzi on, we have been taught that one learns through experience, or through this roll-up-your-sleeves-and-do-it method. And we have certainly learned from military training the necessity for the learner to participate in the process or act which he is learning. These are just samples of the experiences which we found helpful:

One group installed an arterial STOP sign at the front of the classroom, measured back 25 feet and marked it plainly, thereby giving pupils from then on an experience concept with which to estimate and compare all other distances.

Some pupils set up a model of a hydrant, measuring and marking plainly 10 feet on each side, again getting an experience concept which is practical (and money-saving!).

How often we hear a pupil say that there are 3 square feet in a square yard! As suggested in the 18th yearbook (from which we have received a wealth of ideas), one teacher had a square yard painted on the floor, with the square feet in it marked off. The class refers to it *often*. Other teachers have had pupils make models of a linear foot, a square foot, and a cubic foot, and then they keep these three on a handy shelf or radiator cover, and when there is a question about interpreting an answer, they just refer to the three units to see which kind of unit is the reasonable one to use.

All pupils of course discover the relationship between the circumference and diameter of a circle by actual measurement. The Bible reference from II Chronicles 4:2 about a molten sea 10 cubits across and 30 cubits around is always interesting to pupils who have actually measured and found the more exact ratio.

The properties of solids are discovered by all pupils by constructing, examining, studying, comparing, measuring, pondering.

Some algebra pupils have made beautiful transits which the classes have used in practical work around the school yard.

In the study of socio-economic topics, some teachers find that *dramatization* of certain procedures greatly clarifies them, as:

Opening an account at the bank, or
The procedure of getting a loan, or
(For brighter students), The formation of a corporation, or
The travels of a check, or
Where the tax money goes.

Dramatization also of the various plays in THE MATHEMATICS TEACHER has made much more interesting and meaningful some of the history of mathematics.

Some pupils have made interesting collections of geometric figures in buttons, rock crystals, dress materials, necktie materials, tiles, quilt patterns, shells, tapa cloth, Indian work. Development of understandings through such means cannot be measured by that standardized test

at the end of the semester, but nevertheless they are some of the fundamentals.

Algebra teachers have included in their work the simple but fantastically interesting formula of Einstein's for atomic energy, namely $E = mc^2$, where E is energy, m, mass, and c the velocity of light. This is a thrilling glimpse for these students into the dependence of modern science on mathematics.

Relating the mathematics work to the work of other departments such as science, home economics, mechanical drawing, art, shopwork, has been fruitful. However, at times it has done more harm than good. For instance, one day a teacher was trying to teach a class that in order to find what per cent 40 is of 20, one writes 40 over 20 as a fraction and proceeds from there. One boy wanted to write 20 over 40, protesting that his art teacher said that the small things always go on top!

Movies have been helpful, but there are not many goods ones available. I hope that the Committee on Multisensory Aids for the National Council will soon provide us with some first class teaching aids in the form of movies. If there is one thing we have learned from the Armed Forces, it is the importance and efficacy of training aids such as movies, strip films, models, mechanical devices, instruments, tools, charts, mockups. We often hear of things going in one ear and out the other, but never do we hear of things going in one eye and out the other, because visual training sticks.

Supplementing the textbook, we have found the pamphlets put out by commercial companies such as Ford, General Motors, Westinghouse, and the Hamilton Watch Co. helpful.

The pamphlet entitled "The Role of Mathematics in Consumer Education" from the Consumer Education Study, has been very helpful in pointing out what the junior high school can do along this line and in mentioning a wealth of easily available references.

Many wartime materials have served as excellent motivating devices such as Technical Manual 1-900, MATHEMATICS FOR AIR CREW TRAINEES, published by the Government Printing Office.

I must not neglect to mention the advertisements for materials in THE MATHEMATICS TEACHER. Almost every issue contains some new poster, model, pattern, or teaching aid which is helpful.

Through these many mathematical experiences and the study of these applications of mathematics, pupils will be convinced of its *usefulness*, and only then will they apply themselves with the full measure of their abilities. I am reminded of the girl who was neglecting her algebra and was therefore reprimanded by her father.

"The thing you should do," said her father, "is to spend more time on your algebra and less on the boys."

"But tell me, Dad," said the daughter, "which will I use more—algebra or the boys?"

In conclusion, we have made progress—inspired by such educators as Pestalozzi. If we are to do a good job in postwar junior high school mathematics, we need good textbooks, AND teachers who realize that children learn by doing, and who therefore plan interesting, meaningful experiences for pupils in a laboratory setting.

The Case of Decimals Versus Common Fractions

By J. T. JOHNSON
Chicago Teachers College, Chicago, Ill.

WE ARE living in an atomic age where space and time have been reduced and shortened. Simplified practice and speed is the order of the day. We have learned through excursions into the field of chemistry how to make new products in plastics. We have discovered new medicines and new vitamins. We are planning new kinds of houses and have learned new methods of preparing foods. We have added new and safer methods of transportation. We can span our continent in less than 12 hours. We can reach any spot on the globe in 60 hours or less. We have sent rockets 60 miles into the stratosphere and we have even hit the moon 240,000 miles away.

Improvement through simplification is evident on every hand—in industry, in manufacturing, in engineering and technology. But when we come to the curriculum of our schools we find that in many cases we still cling to the antiquated methods of our ancestors of six generations ago.

To quote from a Colonel in the United States Army, "The greatest obstacle to simplifying work has not been created by technical difficulties. Rather, it is set up by the mental attitude of people doing the work, who feel they are already using the best possible method."

Is it possible that there may be some little avenue in the field of the teaching of arithmetic where contents and methods could perchance be improved to keep pace with this ever advancing movement towards simplicity and efficiency?

The writer will take as his hypothesis that there is room for improvement in efficiency in the teaching of arithmetic and that this time in the midst of reconversion on every side is the most opportune time to effect an improvement. This discussion will be confined to the possible improvement in the teaching of decimals and common fractions. Suggestions for improvement will be made in arrangement, grade placement and methods of teaching.

ARRANGEMENT

In arrangement the subject of decimals can and should follow whole numbers and United States money which are studied in the third and fourth grades. After the pupils have been made acquainted with whole numbers so that they can read and write four-place whole numbers and four-place numbers involving dollars and cents, they can through the meaning of cents and dimes and dollars take up the meaning of the easy decimals of tenths and hundreths. In performing operations with these easy decimals the pupils get further practice in the four operations of whole numbers which they have not as yet fully mastered.

In contrast to this our arithmetics which have been chained to the traditions of the past, begin teaching the operations of addition and subtraction with common fractions in the fourth grade before the habit of addition and subtraction with whole numbers has been fixed. The operations in common fractions are so entirely different (as every one knows) from the operations with whole numbers that a confusion is set up in the child's mind and this prevails generally throughout the the fifth grade. Then in the sixth grade the traditional course takes up decimals as a kind of fraction but the operations learned in common fractions do not apply here and more confusion is the result. The principle of place value which is the fundamental principle of our whole number system is the principle underlying decimals. This has not been mentioned or used very much in the study of common fractions for the reason that common fractions have no place value. They exist

as hangers-on at the end of whole numbers but they occupy no place in our number system.

Before decimals can be taken up and understood in the sixth grade where they have traditionally been taught the teacher has to go back and reestablish or more often establish the understanding of place value in the pupil's mind. An undue amount of time is spent on it at this time because of the negative transfer effect from the teaching of common fractions.

Because the common fraction was invented some 3,000 years before the decimal, it has had this tremendous advantage from the point of time in use. Recent studies and experiments, however, have shown that operations with decimals are simpler and easier to use. Therefore the order of historic appearance of these two operations is no longer a criterion for the order of teaching them.

That decimals are more easily learned is attested to by results of an experiment some years ago.[1] A 5B grade that had spent 14 weeks on the meaning of the fifteen most commonly used common fractions and their decimal equivalents and how to add and subtract decimals, was given a ten item test on addition and subtraction of common fractions. A 5A class, one semester older, that had studied common fractions for a year or more was given the same test. The 5B group did it by decimal equivalents and the 5A group did it by the usual method of least common denominators. The results were as follows:

	Group	*N*	*Mean Acc'y %*	*P.E.m*	*Mean Time in min.*	*P.E.m*
Dec.	5B	46	94.3 − 9.2	1.38	6.77 − 1.6	.24
C.F.	5A	39	75.3 − 14.8	2.37	19.63 − 5.73	.92
	Difference		19	PEd 2.76	12.86	PEd .95
	Critical ratio		6.9		13.5	

That decimals are *preferred* to common fractions is shown by the fact that mechanics has decimalized the inch, engineering has decimalized the foot and aviation has decimalized the mile. The writer discovered some time ago that a broker prefers decimals to common fractions. He was computing the price of 20 shares of stock listed at 28⅝. To the surprise of the writer he did it by multiplying 28.625 by 20. The writer asked if he ever used the common fraction in which the prices were quoted in figuring prices. He said he never did because the decimal was easier. To this should be added an interesting testimony from an article in the April 1945 number of the *Journal of Educational Research*.[2]

Due to the fact that paints vary greatly in weight, it is impossible to know just how many gallons of paint there are in a barrel by knowing that the barrel is full. It is therefore the problem of the shipping clerk at the U. S. Paint Company to ascertain the gallonage of each barrel prior to its shipment. His computational work is illustrated as follows:

	Lbs.
Weight of empty barrel	*36*
Weight of full barrel	*481*[3]
Net weight of paint	*415*

The barrel is tagged to the effect that this paint weights 14⅜ lbs. per gallon.

Therefore, changing 14⅜ to 14.375, he divides the net weight of the paint (415) by the gallon weight of the paint (14.375) to get the number of gallons in the barrel (28.8).

Note that the result was even given in decimals, 28.8 gal. when it could have been given as 28⅘ gal.

The order of arrangement, then, should

[1] For full treatment of this experiment see Tenth Year Book, N.C.T.M., "Economy in Teaching Arithmetic," by writer, pp. 121–156.

[2] *Journal of Educational Research*, XXXVIII, Apr. 1945, p. 635.

[3] This should be 451, (451 − 36 = 415) as 415 was used in the computation.

be, *Whole Numbers, United States Money, Decimals* and then *Percentage*. This follows and emphasizes in a continuous and unbroken sequence the principle of place value, each operation building up a foundation for the operation ahead and each operation ahead in turn enlarging, strengthening and reenforcing the principle of place value in the operation before.

Grade Placement

The grade placement depends upon the arrangement of these operations, order of arrangement being the more important. Again, the grade placement cannot be a fixed affair. It depends upon the locality, mental ages of pupils and school environment.

Much informal work should be done in the earlier grades before the number system as a sequential and logical system is taken up. In some localities the formal work may be taken up earlier than in others. If we begin the formal work in 2A or 3B we could easily expect the 390 number facts to be completed by the end of 5B or 5A. Operations with whole numbers could begin in the third grade with increased emphasis in the fourth grade. United States money as dollars and cents could begin in the fourth grade with increased stress in the fifth. The easy phases of decimals could then begin in the fifth with the main emphasis in the sixth followed by percentage in the later grades. This does not mean that two years only is spent on each operation. Each operation after introduced should be continued throughout the eight years but with less emphasis after the first two years. In this way each skill in whole numbers and decimals furnishes an automatic maintenance program instead of necessitating the artificial maintenance drill we find in most textbooks.

Method of Teaching

In teaching the decimal, since it follows United States money and not common fractions, it is introduced as an extension of whole numbers and not as a new kind of fraction. Employing as background material the pupils' knowledge of whole numbers and United States money and the speedometer readings in a car, the meaning of decimals can be linked to the meaning of whole numbers through the value of the digits in the whole numbers. This cannot be done when introduced by means of common fractions because they have no place value.

Three Functions of the Fraction Form

The question now in your mind is, where do common fractions come in? Are they shut out entirely? No, they are coming in all along the line. The first grade child knows the $\frac{1}{2}$ and the $\frac{3}{4}$ from the pies and apples that he has eaten. The common fraction form is coming in for a full share of attention but in a different use perhaps, than formerly.

There are three functions of the common fraction form. They are: 1, the fraction form showing parts of a whole, e.g., $\frac{1}{2}$ or $\frac{3}{4}$ of a pie. 2, the fraction form indicating ratios, e.g., two numbers or two lines may have the ratio 3/5. 3, the fraction form indicating a quotient or as an indicated division, e.g., $A = \frac{1}{2}lw$, or $V = 4/3\pi r^3$.

Our texts in arithmetic have been absorbed almost entirely with the first function and have gone to seed in addition, subtraction, multiplication and division of fractions and mixed numbers. They have spent too much unwarranted time on examples like $6\frac{1}{2} \times 7\frac{1}{3}$ and $14\frac{2}{3} \div 8\frac{1}{2}$. These are never used as such in real life situations.

In his visits to schools in the in-service program this year the writer finds that the pupils are very weak in the understanding of the ratio concept expressed by the fraction form. A question like, What part of 36 is 27? at the beginning of the 7th grade gave 93% wrong answers. Perhaps we should not call these fractions. At any rate it takes two years greater ma-

turity to answer to three-fourths of 36 than to three-fourths of a whole.

This year a series of lessons on the relations of numbers and ratios between them was conducted in the 6th, 7th and 8th grades to supply this felt need. There are just three kinds of these problems. They are: (a) to find a part of a number, (b) to find what part one number is of another, and (c) to find a number when a certain part of it is given, illustrated thus:

(a) ? $\frac{1}{4}$ 24
(b) 20 ? 15
(c) 9 $\frac{3}{5}$?

You all recognize here the basis of the three cases in percentage as indeed they are. They furnish not only the basis for percentage but the basis for a lot of quantitative thinking and reasoning. This kind of work has been sadly neglected in most of our texts. The fraction form as a ratio should be given more emphasis in school curricula than at present.

The fraction form as an indicated division or as a quotient is the last and most difficult concept. It occurs in formulas for the most important use. At a recent convention a speaker showed how this concept should be taught but it is seldom taught as a concept in our schools. She used the illustration of 3 apples to be divided among four boys. She had the apples cut into fourths and as each boy could not have a whole apple there being more boys than apples the first three boys each took $\frac{3}{4}$ of an apple from each of the three apples leaving $\frac{1}{4}$ from each of the three apples for the fourth boy. I dare say that the last boy obtained his share in a unique way and this might have been confusing in forming the concept. The answer, $\frac{3}{4}$, is supposed to show an indicated division of 3 apples among 4 boys but we cannot divide by 4 boys and furthermore, the $\frac{3}{4}$ now is all apple and indicates a part of a whole apple and not an indicated division of 3 apples by 4 boys. This method of explanation may be questionable, depending on what is to be taught. The above demonstration could very well be used for teachers to show that we should not always label numbers in computation except in the final answer.

Summary

To summarize the ideas set forth in this discussion, let it be suggested that we teach decimals as a continuation of whole numbers and United States money using the concept of place value throughout.

If common fractions as part of units are to be added or subtracted let them be limited to halves fourths and eighths to be done orally. If they cannot be added or subtracted orally then the child does not understand them and should wait till he does. Any other fractions as parts of units that cannot be added or subtracted orally should be eliminated from such use and their counterparts be treated decimally.

Eliminate such unnatural operations as $\frac{1}{3}+\frac{1}{7}$, $\frac{7}{8}-\frac{2}{5}$, $7\frac{1}{3}$ plus $6\frac{3}{4}$, $2\frac{3}{4}\times 5\frac{1}{5}$, $6\frac{2}{3}\div 8\frac{3}{4}$ and the like. Children *have* learned these but at great expense and as mechanical tricks soon to be forgotten because never used in later life.

Spend the time formerly used on the above in teaching relations of numbers and their ratios and factors employing the common fraction form for this. Using ratios in finding numbers and finding ratios between numbers is very valuable work in that it not only furnishes food for quantitative thinking but it also builds a backlog of concepts that will bear fruit later on in percentage, and percentage is now used by almost every one.

The Review versus the Telescoped Reteaching of the Work of Preceding Grades

By R. L. Morton
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What is a Review?

Etymologically speaking, to review means to view again. But the second view may be the same view repeated or a different view. The Funk and Wagnalls dictionary calls a review "a second, repeated, or new view." For teaching purposes, there is a vast difference between a *repeated* view and a *new* view. This difference concerns us here.

If the review is a repeated view, it may well be precisely the same view that was previously experienced. To use a less complimentary expression, it may be merely a rehash of a former experience. But if the review is a new view, it may be experienced from a different vantage point, thus repeating in part but adding to or supplementing the earlier view. For example, I once had an interesting and stimulating review of the Grand Canyon of the Colorado River from the north rim after having previously viewed it from the south rim.

The Nature of Reviews in Arithmetic

Arithmetic reviews too frequently are merely a rehash of previous experiences. It is assumed, and not without reason, that the earlier experiences have not been adequate, or of the proper kind to produce satisfactory results. Hence, the teacher rolls up her sleeves, as it were, and begins the year's program with a review. What kind of a review does she provide?

To be more specific, let us suppose that a review is being provided at the beginning of the program for the fourth grade and that the review is to be concerned with the process of addition. A common practice is to begin by giving the pupil a test on the "100 addition combinations" or the "100 addition facts." The very commendable purpose is to find out which of these, if any, the pupil does not know. If some of the facts are not known, various devices are employed for practicing or drilling on those facts.

Next, the pupil moves to column addition, without carrying. He may or may not be given a hint to the effect that higher-decade combinations will appear in these columns. At any rate, he practices adding columns. If his answers are incorrect or if he conspicuously fumbles the job, he is given additional drill on adding columns of numbers.

Carrying in addition is next on the scene. There is a test, there may be a model example to illustrate the process, and then there is further practice.

Now this kind of review program is the kind which is carried out in many schoolrooms because it is the kind which is suggested by the course of study or other aids at the disposal of the teacher. It leaves much to be desired. It fails seriously to accomplish the major purposes of a review. Its principal faults may be stated briefly as follows:

1. The test and drill kind of review makes no provision (or at least inadequate provision) for the pupil to learn that which he was previously failed to learn or has learned and forgotten. What such a pupil needs is a chance to learn. Practice should come later to fix what has been learned and to maintain it at a high level of working efficiency.

2. Because no provision has been made for learning that which has not been learned or has been forgotten, this kind of review encourages the pupil to discover and use undesirable methods and to habituate himself in them. Many pupils

in the fourth grade need to learn higher-decade addition in spite of the fact that this has been taught in Grade 3. If they have not discovered the relation of the higher-decade combinations to the basic facts, they will probably use some circuitous method, such as counting, to add 4 to 18 in the example 4+8+6+4=? Some pupils are even led by their teacher or their own misquided efforts to think of 18+4 in such an example as a situation requiring carrying. Of course, there is no carrying in higher-decade addition. The individual who has really learned to use higher-decade addition with bridging in columns, will add 4 to 18 *as a single act of thought*, as it were.

3. The test and drill kind of review fails to increase the pupil's understanding of the process. As processes and phases of processes are rationalized in one grade after another, the rationalization should be at ever higher and more mature levels. Even though the pupil understands the explanation of carrying in addition as illustrated with coins in Grade 3, he may not understand equally well the explanation in terms of the ones, tens, etc. of our decimal number system for the latter is more abstract and more difficult. His understanding of carrying should grow and increase as he matures and should be lifted to a higher level in Grade 4 than in Grade 3.

The test and drill kind of review usually makes little use of rationalization after the first presentation of a topic. The work tends to degenerate into mere mechanical repetition.

4. The test and drill kind of review fails to take advantage of certain very important generalizations which should have been developed when the topic was first taught and which certainly should be taught when the topic is reviewed. For example, there may be given in Grade 4, again in Grade 5, again in Grade 6, again in Grade 7, and again in Grade 8 a test on the entire 100 basic facts in addition, *including the zeros*. Now, in learning the zero facts in addition, the pupil should have learned that the sum of zero and *any* number is that number. Following this, he should have had practice on the zero facts only in situations in which they would normally occur and not separately or in isolation as is so often the case in reviews. But if the zero facts have not been taught and practiced in this manner in preceding grades, certainly the review should stress the generalization stated above and should supply practice exercises which are in harmony with the uses of zero facts in life experiences.

It is surprising that more progress has not been made in the direction of teaching the zero facts of addition, subtraction, multiplication and division by generalization and of limiting the practice to lifelike situations. It is not difficult for a pupil to learn that the sum of zero and any number or of any number and zero is that number. The other generalizations are also easy to learn; that is, if zero is subtracted from any number the remainder is that number, or if any number is subtracted from another number just like it the remainder is zero; also, if zero is multiplied by any number or if any number is multiplied by zero the product is zero; and finally, if zero is divided by any number the quotient is zero. The multiplication and division generalizations are especially easy if our language for the product of such a combination as 0 and 4 is "zero 4's" and "four 0's" instead of "zero *times* 4" and "four *times* 0" and for the quotient of 0 and 4 is "How many 4's in 0?" instead of "4 *goes into* 0 how many times?" or "0 *divided by* 4 equals what?" or "4 divided into 0 equals what?"

To ask a pupil on a review test to answer such a question as "What is the sum of 0 and 4?" or "If 0 is subtracted from 4 what is the remainder?" or "if 4 is subtracted from 4 what is the remainder?" is to imply that the pupil cannot respond correctly to the related real life situations unless he "knows" these zero facts. This kind of review and original teaching im-

plies that the pupil needs such experience in order to be able to answer correctly the question, "If Bill had 5 cents and spent nothing how much did he have left?" or "If Bill had 5 cents and spent 5 cents for an icecream cone how much did he have left?" In the former case, presumably he is supposed to think, "5−0=5, so Bill had 5 cents left"; and in the latter case, "5−5=0, so Bill must be broke." Obviously, what I have described occurs nowhere but in a schoolroom or an insane asylum.

The Telescoped Reteaching Plan

Many if not most pupils need more than a review at the beginning of any year after the organized program of arithmetic instruction has been begun. They need a chance to *learn* or *relearn*. They need also an opportunity to *learn more* than they have previously learned, to gain a greater degree of understanding of numbers and the processes with numbers. They need a program which we are here calling "telescoped reteaching."

Why we say "telescoped reteaching" will be obvious. We say "reteaching" because there is an abundance of evidence supporting the contention that much learning is at first partial and incomplete; that pupils differ greatly in what they acquire from their experiences in previous years; that there is considerable forgetting, especially during the long summer vacations; and that more mature degrees of understanding become possible as the pupils become more mature.

We say "telescoped" because, obviously, the reteaching can be briefer and less detailed than was the first teaching. The reteaching of a topic common to all of these grades, such as addition, will be briefer in Grade 4 than in Grade 3, briefer in Grade 5 than in Grade 4, briefer in Grade 6 than in Grade 5, etc.

The Nature of Telescoped Reteaching

To illustrate what is meant in this discussion by "telescoped reteaching," let us outline a suggestive program for teaching the addition and subtraction of whole numbers in Grades 3–8. A similar program may be outlined, of course, for the multiplication and division of whole numbers, for common fractions, and for other topics, although not all will be begun in Grade 3.

In Grade 3, we shall teach the 81 addition facts and the 81 subtraction facts without zeros, the addition and subtraction of two-place numbers and three-place numbers without carrying or borrowing, the use of zeros in the addition and subtraction of two-place and three-place numbers, higher-decade addition without and with bridging and its use in adding columns, carrying, borrowing, checking, and the terms *addition fact*, *subtraction fact*, *plus*, *minus*, *sum*, *difference*, and *remainder*. Other matters to occupy our attention in Grade 3 will include an acquaintance with one-place, two-place, and three-place numbers, the meaning and use of certain very common fractions, measures, and a beginning in multiplication and division. It is estimated that the addition and subtraction program which has been outlined will require about 65 per cent of the school year. Of course, tests, maintenance exercises, and applications in problems are included.

In Grade 4, we shall set up a very definite plan for telescoped reteaching of the addition and subtraction which was taught in Grade 3. We shall re-present all of the 81 addition and the 81 subtraction facts. Instead of re-presenting them all at once in a test, we shall take a few at a time, the easiest first, and show the relationship existing among the addition and the subtraction facts which come from a pair of one-figure numbers. The plan will give us a chance to reteach the items which need to be retaught as we go along and to present a meaningful second view of the addition and subtraction which was taught in Grade 3. The reteaching of the addition and subtraction facts without zeros, together with their use in problems and in single columns in which no higher-decade

addition is involved will require, we estimate, about 4 per cent of the days in the school year. Of course, the plan is flexible and the teacher should have much discretion as to the amount of time to be used.

Then the other elements of addition and subtraction which were taught in Grade 3 are retaught. The entire time requirement for reteaching addition and subtraction in Grade 4 is estimated to be 16 per cent of the school year. That which required 65 per cent of the time in Grade 3 for teaching requires approximately one-fourth as much time in Grade 4 for reteaching. The work in addition and subtraction is then extended to include more difficult examples.

In Grade 5, the reteaching plan will include that which was taught in Grade 3 and retaught in Grade 4 and also that which was taught in Grade 4. So far as the addition and subtraction of whole numbers are concerned, it is estimated that the time requirement for reteaching will be about 8 per cent of the school year. This shows a considerably greater degree of telescoping in Grade 5 than in Grade 4 but far more than the perfunctory review of addition and subtraction which often characterizes the program for the early part of Grade 5.

In Grade 6, there will be still further telescoping of the reteaching of addition and subtraction but it is obvious that the amount of time used for this purpose cannot be greatly reduced below that used in Grade 5 without losing the benefits of the reteaching plan. It is suggested that the reteaching of the addition and subtraction which has been taught in Grades 3, 4, and 5 can be done successfully in Grade 6 in about 6 per cent of the time of the school year.

In Grade 7, 5 per cent of the time will be adequate for the reteaching plan which has been indicated for these two processes with whole numbers. By this time, there is ground for the expectation that the addition and subtraction processes will be well under control. Consequently, in Grade 8, except in unusual cases, very little further attention need be given to these processes beyond the provision of an adequate amount of maintenance material for practice.

The Use of Diagnostic Tests in Telescoped Reteaching

We have spoken somewhat disparagingly of what we have called "the test and drill" kind of review. This should not be interpreted to mean that we are condemning either tests or drills. If tests are diagnostic in character, they serve a very real purpose in the telescoped reteaching plan. Diagnostic tests should be keyed so that the parts of the process on which the pupil needs further instruction are clearly indicated. The pupil should receive the required further instruction before he engages in more drill.

Drill is important and valuable as a means of fixing what has been learned and maintaining skills at a satisfactorily high level of usefulness. But the meaning of number and the processes with numbers is not attained through drill. Drill should come after the development of an understanding, not before.

In summary, it is suggested that the conventional review in elementary school arithmetic is inadequate. It fails to locate specific weaknesses in pupils and to provide the necessary relearning opportunities. It encourages mechanical manipulation rather than the intelligent use of number. It fails to provide at ever higher and more mature levels the rationalization of processes and phases of processes. It is proposed that the conventional review be supplanted by a program of telescoped reteaching.

Mathematics for the Non-Academic Student

By Anthony Marino
Youngstown, Ohio

On December 1, 1945, the Ohio Department of Education made public the results of a test given to 2,304 selected seniors from Ohio High Schools—public, private, and parochial. The tests indicate that many Ohio High School seniors are woefully weak in simple arithmetic and are ignorant of much of the basic general information they should have mastered before the eighth grade. Education Director Clyde Hissong, in discussing the above results, asserted that "too many Ohio school children just don't know the answers." He also indicated his suspicion that perhaps the teacher training schools and their products are at least partly to blame for the situation.

Placement tests given by Ohio State University likewise indicate definite arithmetical deficiencies. Reporting on the results of tests given a large number of freshman students, Professors Alfred B. Garrett and Harold P. Fawcett assert that mathematical weaknesses of high school graduates in general are such that they will hamper the success of any person regardless of whether he goes to college or directly to work. In addition the professors offer several suggestions for eliminating the obvious deficiencies which exist in the mathematical preparation of high school students, particularly: 1. Lack of facility in handling simple mathematical processes; 2. Lack of ability to solve problems.

The first of their suggestions is a refresher course involving the fundamentals of arithmetic to be given all high school seniors who on examination show a deficiency. The second solution, one of long range value, is a more careful check on proficiency in arithmetic for the certification of grade-school and high school teachers. The final remedy is in the teacher training program. "A comprehensive test for all prospective teachers before certification would very probably eliminate many teachers who would otherwise carry into the classroom the same deficiencies with which they graduated from the eighth grade or from high school."

Do the above indictments and remedies apply only to the students and teachers of Ohio? Of course not. One needs only to refer to the experiences of the Army and Navy Departments in their service and special schools. These indicate that arithmetic illiteracy exists everywhere. What is the answer? Perhaps the answer is to be found in the statement of Professor William A. Brownell of Duke University that "arithmetic programs of the past 25 years have given children little chance to use ideas and skills already learned in solving their personal problems and they have neglected the meanings and rational principles which make arithmetic a phase of mathematics. That this second deficiency is becoming steadily more recognized is attested by the frequency with which we hear that arithmetic must be taught meaningfully."

Many of us are aware of the fact that a large number of our pupils are not furnished with the fundamental training in arithmetic that is valuable and essential whether they continue their work in school or are obliged to begin active service in the community. Also we realize that our present high school mathematics does not meet the needs of a large per cent of high school pupils, pupils who are either incapable of profiting from the study of these courses, or who, if capable, would profit more from another type of course. In other words we must give serious thought to the need of providing satisfactory courses in mathematics for those who are not preparing for entrance to college.

Before presenting my ideas on the type

of mathematics that I believe will function in the everyday lives of the boys and girls not preparing for entrance to college, let us consider the popular conception of education and how we struggle to achieve same.

According to our leading educators the aim of education is to train the youth to earn an hornorable living; to provide them with the means of utilizing their ever increasing leisure; and to train them to become worthy and noble citizens.

The achievement of this aim is a tremendous task. No one man, no one church or sect, no one society can accomplish it. Everywhere during the last century this task has been left to each nation as a whole represented by its government. Professor Cubberly in his discussion of the topic "Education Becomes a National Tool" writes, "Today progressive nations everywhere conceive of education for their people as so closely associated with their social, political, and industrial progress, and their national welfare and prosperity that the control of education has come to be regarded as an indispensable function of the State."

Simultaneously with this general acceptance of the state function conception in education, a tremendous advance in scientific knowledge has taken place. Although an acquaintance of mathematics and science prevailed in Caesar's time, the Romans had not learned how to apply their abstract knowledge to any scientific method for investigating the phenomena of the material world. The same observation applies to the world that existed at the time our country struggled through its formative period. Indeed very few changes in living were effected from the time of Caesar to the Revolution of 1776. No wonder that George Washington commanding an army many centuries removed from Caesar's time could not transport or move troops any faster than the latter. Indeed we may say that conditions under which these two men lived were essentially the same. If Washington could return to our world of railroads, steamships, electricity, airplanes, and radios he would be almost as bewildered as the great Roman himself.

When our forefathers in Europe and in the United States began to apply their scientific knowledge, inventions resulted which eventually transformed living conditions. Gigantic advances were made in the field of chemistry, physics, physiology, and the other related applied sciences. These were applied to inventing machines, appliances, and devices to save time, increase wealth, and bring the world closer together. As these advances progressed and leisure and ease became the possession of the Americans, a new type of education was needed. Railways, steamboats, trades, telegraphy, the introduction of steam and electric machines in the factory, farm, and home compelled us to reorganize our methods of education.

Even though this transformation progressed rapidly the old curriculum persisted to the end of the nineteenth century. Indeed a few of the old traditional subjects have been carried over even though they do not seem compatible with our present conditions. As long as the U. S. had a frontier where new sources of wealth were always available the problem of making a living was not difficult. With the disappearance of the frontier, life has become harder and the task of educators has become more difficult.

Everyone realizes the need of studying our whole system of education and changing it to suit the present and new era. In revising our courses of study to meet the new status, we should give more attention to the mathematics that must be pursued by those who are preparing to enter industry and business. The mathematics course for these should be practical and compatible with the aim of education. They should help enrich the curriculum in providing a better guide to citizenship. This does not mean that the mathematics for college entrance should be decreased. Besides algebra and plane geometry, solid

geometry, college algebra, and trigonometry should be required by those who are pursuing an academic course.

Besides enriching and revising the curriculum better courses of training for teachers must be provided for. Our teachers training institutes and our administrators must eliminate what seems to be a long and continuous period of experimentation; a period wherein so many educators set forth their particular ideas as the panaceas for all the evils that exist in education.

During the last few decades our educators have been very busy inaugurating various schemes of school regulation. Some sort of scheme or arrangement for accelerating or enriching the work of the schools has been devised by practically every modern institution devoted to the training of teachers. While some of the methods proposed or actually in use meet individual differences—to most of us they are bewildering.

The high school teacher has listened very attentively to educators evaluating many methods of teaching; the Project Method; the Morrison plan; the Socialized Recitation Plan, etc. After studying a number of these, some of us have reached the conclusion that educators to whom has been entrusted the task of preparing college students for high school teaching either have no definite plan or can not reach a conclusion as to which method is the best.

At the present time, we have a number of teachers in the Youngstown system trying to qualify for a Master's degree in Education. Many of them feel that to a certain extent they are actually buying a degree. They complain their undergraduate courses in education were either worthless or the advanced courses which they are now pursuing are repeating to a large extent their previous studies. Then too, they find that frequently the titles and descriptions of courses in education given in university catalogues are misleading. To illustrate Education 302 as given by one university although it bears no resemblance to course 207 as described in the catalogue of a second university really overlaps it in many essential features. In fact it is possible to observe a great deal of repetition and overlapping in many of the advanced courses in education offered by the same institution. Is it any wonder then that the outstanding criticism is in behalf of better preparation of teachers. Again teachers who have studied various courses in different schools find after presenting their credits to certain institutions for evaluation that they can receive no credit for their work because of some foolish residence requirement.

Research departments of industry and business do not evince any uncertainty or indefiniteness in the merchandising plans they follow.

For several years the writer represented one of the divisions of General Motors as an automobile dealer. From time to time our firm received from the Sales development department of this concern definite and specific sales instruction and procedure. The suggestions presented in their sales manuals certainly did not display the conflict of theories, the indefiniteness that characterizes the work of our colleges of education training students for high school teaching. The time has come for appointing committees for studying the whole system of teachers training and changing it to suit the present and new era.

Besides better courses of training for teachers it should be the duty of administrators and principals to see to it that those who are entrusted with the task of teaching arithmetic to our elementary students have mastered the fundamentals and have had the training necessary for teaching the subject properly. In other words we should put into effect a more careful check on proficiency in arithmetic for the certification of grade school teachers. Furthermore elementary school instructors should see to it that the aims and objectives of arithmetic are realized. This means that pupils should be taught

arithmetic daily whether they are in the mood or not. It may mean also that teachers will be occupied with the task of eliminating deficiencies in simple arithmetic instead of pursuing pet projects to their conclusion. Obviously the above suggestions and remedies apply to junior and senior high school teachers as well.

We are agreed, I believe, that our arithmetic courses for the seventh and eighth grades should provide pupils with training that will be valuable and essential whether they continue their work in school or are obliged to begin active service in the community. Further it is agreed that "arithmetic must be taught as a system of ideas, to be mastered by insight and meaningful applications."

Even though these aims are carried out there will be pupils entering the ninth grade with pronounced mathematical deficiencies. These, however, should be limited to boys and girls who tried and were promoted because they lacked the ability to acquire proficiency in arithmetic. The numbers so rewarded should be limited.

Sometime ago a committee was selected to study and revise the mathematics curriculum of the Youngstown Junior-Senior High Schools, grades seven through twelve. Besides serving the committee as its chairman, I accepted the task of preparing a manual for the use of teachers in our school system. At this point I can do no better than to repeat the statements relating to Mathematics for Non-Academic Students which appear in the Youngstown Manual.

Mathematics for Non-Academic Students

The Curriculum Committee entrusted with the task of revising the study of mathematics in Youngstown's Junior and Senior High Schools are of the opinion that all students in high school be required to study mathematics at least two years.

The committee is aware of the high percentage of failures in elementary algebra. Unquestionably many of these are due to the fact that a large number of the pupils are incapable of doing successful work in formal mathematics. We recommend, therefore, that an aptitude test be administered to all pupils in 8A arithmetic. The purpose of this test is to discover those boys and girls who are not capable of profiting from the study of algebra and thus spare them the waste of a semester or more of time and effort. Pupils whose test scores indicate that they have superior ability in mathematics should be advised to elect algebra whether they enroll in the academic courses or not. After all this subject is well suited to their capacities and interests whether they are preparing for college or for entrance into active service in the community. On the other hand, pupils whose test scores indicate that they are unlikely to succeed in algebra should be advised to take social mathematics instead of algebra.

Ninth Grade General Mathematics

During the last decade many high schools have introduced courses in practical or general mathematics. In most places ninth grade students not enrolled in algebra were permitted to select the new course. Simultaneously several new text books were introduced to meet the new requirements. It was hoped that these would provide the students the instrument which would enable them "to carry over their mathematical training into their later life and develop those qualities and ideals which will better fit them for the life they are to live."

Most of the new texts were written by men who had previously published successful algebra and geometry text books. Collectively the books on general mathematics contain essentials of arithmetic, graphs, scale-drawing, intuitive geometry, algebra, etc.; individually they manifest, comparatively speaking, a minimum agreement as to the material that is needed to aid pupils in acquiring the processes that are to be used in their later everyday ex-

periences. In spite of the wide divergence of opinion as to the subject matter the prefaces of the new texts nevertheless proclaim the same aims and objectives.

In Youngstown we have already tried two of these and both have been considered unsatisfactory. The second one was chosen by a committee of which the writer was a member. We examined and evaluated all the new text books and in addition gave representatives of various book companies the opportunity to present the merits of their publication. Later a vote was taken. The result: nine votes, nine different choices. It was then decided that each indicate a second as well as a first choice. The book receiving the largest number of first and second choices was adopted.

It should be pointed out, however, that there are too many classes in practical mathematics composed of pupils illiterate in arithmetic. In fact there exists the tendency to make these classes the "dumping ground" for the maladjusted. Hence they are avoided by teachers. The result is that they are frequently taught by members of the faculty actually antagonistic or by teachers not trained to handle arithmetic.

Aims and Objectives of General Mathematics

Courses in general mathematics should be practical and compatible with the aim of education and should help provide a better guide to citizenship. They should take into consideration the fact that many students enrolled in these courses are incapable of profiting from the study of algebra. The topics studied, as well as the amount of time spent on each, should consider the needs and interests of the pupils involved.

General or social mathematics should be devoid of algebra, geometry, graphs, scale-drawing etc. Why spend valuable space and time on a mere introduction to various items of academic mathematics which require students months to master? Why not devote this space to material stressing the place of mathematics in real life and modern situations. In other words why not provide a real attempt to correlate processes in arithmetic with our social problems.

The primary aim of general or social mathematics is to teach students the skills, abilities, attitudes, appreciation, and habits that will be of value to them in everyday life situations, thus making them more efficient and intelligent members of society. Obviously practice in the fundamental processes should have a prominent place. Furthermore the tools necessary for solving problems of present and future individual and significance should be provided. In other words the material used should in one way or another relate to everyday living. It should be meaningful. Although the contents are to be highly socialized at no time should we lose sight of the fact that "arithmetic must be taught as a system of ideas, to be mastered by insight and meaningful applications. It never must be taken for granted, but rather reviewed constantly."

We believe the table of contents that follows provides a course that is in accord with the aims and objectives enumerated above.

Part I. Review of Elementary Mathematics

1. Review of Arithmetic
2. Everyday Uses of Per Cent

Part II. Solving Practical and Essential Problems

3. Home Problems
4. Installment Buying
5. Automobiles and Mathematics
6. Mathematics in Aviation
7. Insurance
8. Social Security and Unemployment Compensation
9. Problems of Population
10. Taxes
11. The Problems of Peace
12. Supplementary Exercises and Problems

Details Concerning the Contents of the Recommended Course in General Mathematics

Recently the writer received a copy of "The Role of Mathematics in Consumer Education," developed by a committee representing the National Council of Teachers of Mathematics. The comments concerning the mistakes made in teaching the mathematical topics of consumer education to high school students interested me very much, particularly the claim that we have failed "to emphasize the social implications of such concepts as taxation, insurance, compound interest, and installment buying, probably for the reason that we have done too much computing. It is important for the pupil to understand the concept of taxation—to get the idea that taxation is a co-operative device enabling the citizen to buy valuable public services; schools, road, parks, protection to health, delivering of mail, etc.—that he could not possibly afford if he were acting alone. It is far more important that the pupil get this broad social implication than it is for him to compute the tax due in each of a dozen situations remote from his experience and interest."

The writer sensed the above mistake some time ago and for a number of semesters has been experimenting with material that should be of value in meeting the above mentioned criticism. In consequence the writer has developed units on the items which we believe should be included in general mathematics. These units emphasize, not only the social implications that functions in the students everyday lives, but also provides the necessary emphasis to computation as a means of obtaining a clear understanding of the principles involved in each unit. Throughout the course selected review exercises are provided for. These are planned to help the student maintain and improve the skills he or she has mastered. Then too each unit contains vocabulary studies entitled "Some New Words." Each of these directs the attention of the student to words that play an important part in his or her newly discovered world. Also each unit contains discussion exercises involving the social problems incorporated in the material of the unit.

This emphasis "on doing" is the natural consequence of the writer's long experience as a teacher of freshman students in three large four-year senior high schools. This experience has convinced the writer that the best way a student can obtain facility with mathematical relationships is by working a large number of problems involving the same operation. Hence these units collectively contain more problems than any other of a similar nature.

The discussion provided in each unit plus the large number of thought problems dealing with the personal needs encountered in our everyday experiences provide the student with a worth while preparation, regardless of his future occupation. In other words the student obtains a great deal of information concerning our everyday affairs.

The course is divided into two parts. The first part is a review of arithmetic. This review is conducted by means of tests. Timed tests are provided for each important phase of number relationship. In the second chapter every possible opportunity is utilized to furnish problems of a practical nature. The three classes of per cent problems and such items as profit and loss, commission and brokerage, simple and compound interest, etc. are presented in an informational manner. Wherever the opportunity presents itself in this and the units that follow business approaches, methods, and procedures are made use of.

The second part, entirely practical in nature, is devoted to home and social problems. Its contents comprise three-fourths of the course.

In order to realize the objective of each unit continuity is emphasized. For example, all types of interest charges; installment buying in automobiles and electric appliances; personal loans; home loans; G. I. Loans; and bank notes are in corporated in the same unit entitled "Installment Buying." The large number of prob-

lems the student works as he or she progresses through the various forms of interest charges, from one type of installment buying to another should give each an appreciation of the objective of the unit; namely, how costly buying "on time" is.

Unit V

A detailed description of unit V entitled "Automobiles and Mathematics" is presented to demonstrate how social implications can be emphasized without neglecting the computation necessary for aiding the student in maintaining and in proving the skills previously mastered.

The unit begins with an account of the importance of the automobile in our everyday affairs; how the auto industry acquired first place among all industries; how the increased use of autos provided employment directly and indirectly to 10% of the wage earners of the United States (before World War II). Indeed prior to "Pearl Harbor" the automobile industry provided the largest single market for rubber, gasoline, steel, lubricating oils, plate glass, upholstering, leather, etc. In addition a larger percentage of copper, zinc, cotton, wood, and other commodities were consumed by the motor industry.

An industry so vast and a commodity so extensively used and so much desired by every inhabitant, obviously presents material for many problems of a practical nature.

The first problems contained in the unit relate to depreciation. Each year the makers of the automobile present new models involving improvements in speed, dependability, safety, convenience, and appearance. These yearly improvements and refinements stimulate the interest in and the desire for new automobiles and in consequence the value of cars in use depreciate rapidly. The depreciation of an auto is not uniform over a period of years; that is, depreciation is greater during the first year than in subsequent years. This discussion indicates how depreciation in any district is determined. In addition the student is introduced to the "blue" and "red" books, the tools used by dealers in appraising cars.

The next group of problems deal with interest; interest on the money invested in an automobile as well as the cost of financing. These items, of necessity, are included in exercises based on the question "The Cost of Owning a Car." This list of exercises is followed by problems dealing with Gasoline and Oil per mile. Other items such as license plates, repair bills taxes, insurance are then introduced and a number of problems are presented dealing with the "Cost of Operating a Car per Year." This naturally leads to the question of "How Long One Should Keep A Car Before Trading It On A New One."

The operation of a car involves instruction in "traffic safety." This provides the writer with the opportunity of presenting statistical information concerning our death, injury and accident totals. It also enables the writer to emphasize safety and the need of uniform traffic laws. By utilizing discussion questions and problems involving traffic accidents the student is shown the importance of traffic safety and the slogan "Don't Take Chances."

It was natural that our leading industry should become America's No. 1 defense industry during World War II. This provides the opportunity of presenting numerous facts concerning the magnitude of the contribution made by the auto and the auto industry to our war effort.

Finally the unit provides material relating to the question "To-morrow's Auto." This is followed by exercises dealing with the social problems touched upon in the unit. A feature of the chapter is a vocabulary study entitled "Some New Words." This list directs the attention of the student to words that play an important part in the reading material included in the unit. In other words in addition to approximately seventeen pages of reading material the chapter entitled "Automobiles and Mathematics" contains over 100 problems, many of a verbal nature. Can we teach social implications and not neglect computation? I believe we can.

◆ THE ART OF TEACHING ◆

Methods Used in the Selection of Pupils for the Study of Algebra and Geometry in Cleveland

By ONA KRAFT
Collinwood High School, Cleveland, Ohio

EDUCATIONAL psychologists may still be disputing about the kind and amount of transfer of training in which we dare believe, but we know now that the sheer struggle to master something difficult and distasteful has in itself no educational value. Although we recognize the benefits of the study of algebra and geometry, we are convinced that not all pupils are capable of obtaining them.

Who then is to say which ones are capable, and using what shibboleth? By letting the pupil choose? By the mental test results? By the recommendation of the previous mathematics teacher? By the pupil's general average of achievement?

Even with conscientious advisors assisting, the pupil's choice is often unwise. Trivial considerations influence him unduly, hearsay about teachers and subjects. In the May 1935 MATHEMATICS TEACHER H. D. Richardson of Highland Park, Illinois, in a study concerned with predicting achievement in plane geometry, concluded that mental test ratings alone correlated only 50 with geometry achievement grades. Teachers' recommendations alone do not work too well. One person's opinion, however careful he tries to be, is apt to be inaccurate. The previous scholarship record in all subjects is perhaps the best single criterion, although by no means infallible. To combine these methods is more effective than to use any one of them alone.

In Cleveland, as in a number of other places, we make use of one more criterion, the Iowa Algebra and Geometry Aptitude Tests. They were selected by the director of mathematics for city-wide use. The California Aptitude test and the Orleans Prognosis Test are also well-known.

About the fifth week of the semester the algebra aptitude test is given to the second term eighth grade arithmetic pupils, and the geometry aptitude test is given to the second term ninth grade elementary algebra pupils. This is early enough in the semester to have the results available for intelligent use.

The algebra aptitude test consists of examples in abstract computation, simple algebraic procedures, number series, and interdependent changes of values in equations. It is easily and quickly scored with a stencil-type key. It tests the abilities which are the basis for learning algebra. The geometry aptitude test consists of exercises in the reading of geometry content, simple algebra equations, verbal problems, exercises which test the pupil's ability to visualize geometrical figures. It too is easily scored by a stencil key.

Convincing figures of the validity and reliability of these tests have been worked out by their authors and are found in the Examiner's Manual. While I have great faith in statistical measurements, I am trying here to present the case from the standpoint of the Cleveland teachers who are using the tests, rather than from a statistical standpoint.

Originally the pupil's score on the algebra aptitude test was added to his PLR to form his algebra aptitude index. Cleveland calls its mental test ratings PLR's, Probable Learning Rates, because they

are the results of group tests, rather than I.Q.'s, the results of individual Binet tests. In 1944 our Bureau of Educational Research made a careful study of the correlation between the achievement grades in algebra on city-wide tests and the aptitude indexes, and between achievement grades and the aptitude test scores alone. The conclusion was that adding the PLR to the aptitude test score did not increase the correlation between the prediction and the achievement. Since then the test scores alone have been used. A similar study of the geometry aptitude index has not yet been made, and we still add the PLR to double the test score to compute the geometry index.

The director of mathematics makes the following recommendations for all Cleveland schools: 1. Pupils with a Geometry Aptitude Index of 180 and above are to be counseled and urged to take geometry, 2. Pupils with a Geometry Aptitude Index of 145 and below should not be enrolled in plane geometry classes, with the exception of those pupils who, after conference with their parents, decline to follow the advice of counselors. 3. For counseling the middle group, counselors must seek additional evidence, such as previous success in schoolwork, especially in algebra. In general, in the Index range between 180 and 145, those pupils with the higher Indexes will have a greater chance to succeed in plane geometry than those in the lower range. However, there will be exceptions. The pupils who should be excluded are in the lowest 10%.

The recommendations about algebra are in almost the same words, except that in the *surely* group the aptitude test score is 55 and up: in the *positively no* group it is 37 and below; and the *maybe* group goes from 37 to 55. The recommendations about taking algebra and geometry must accompany a pupil who changes schools, and the new school is bound to honor them, unless the pupil himself proves that they have been wrong.

My own school runs from 7B to 12A, with many new pupils in the ninth and tenth grades, and for this reason I am in a position to see the whole picture more clearly than some of the other teachers, for we give both aptitude tests, and teach both subjects. I find that almost everyone who should take algebra is actually taking it. Those who do not want algebra or who accept our advice not to take it usually prefer some other subject than mathematics. Those who take algebra against our advice are usually boys who want to be technicals. Ninth grade algebra is required of all technical boys. Our technical schools are not in any sense trade schools, and those who fail in algebra and geometry have little chance in the other subjects required, but if their parents insist, we must allow them to try.

Some schools offer a course in ninth grade general mathematics as a substitute for algebra, the outline coming from the director of mathematics. In my school we follow a different plan. Those pupils who come to junior high school on account of their age, but who may be as low as third or fourth grade in arithmetic, are kept together in class. They take arithmetic in the seventh, eighth, and ninth grades, instead of for only two years. After the ninth grade, some leave school, and the others are not much interested in mathematics, but their arithmetic teacher sends them to me if they are, and I give them the aptitude test and recommend them or not on the same basis as others. If they take algebra at all, it is in the tenth grade.

The per cent of those who choose geometry is not as great as the per cent of those who choose algebra. In the commercial courses, geometry is not offered, a commercial substitute often given in its place. Girls in technical courses spend all their time in other subjects. But all boys and girls in academic courses and boys in technical courses are either required or urged to study both algebra and geometry, if they are capable. Substitutes for geometry may be some form of social mathematics, or commercial mathematics, or

shop mathematics. Each school meets its own problem. At my school, no girls need a geometry substitute, and we give the boys a year's course in shop mathematics, planned by the director of mathematics. The pupils in these classes are not all inferior. Some would have been assigned to geometry if they had wished.

After I was assigned this subject, I wrote to every senior and junior high school, and to all elementary schools having an eighth grade, asking for the teachers' honest opinion of the value of the aptitude tests. Answers came from almost all, some quite brief, some at length, and several with statistical studies. This cooperation not only makes me most grateful but confident that I can really speak for the Cleveland teachers.

Do the teachers themselves approve of the aptitude tests? Yes, almost unanimously.

Do they approve of the three groups: the *surely*, *maybe*, and *no* groups? Yes, they say it is a good division.

Do they feel that aptitude tests are a help in counseling? Yes. Almost every school uses all other available information, too. If the guidance in a school had been superior before we began using the tests, and if it had developed its own tests to choose the pupils to take algebra and geometry, the aptitude tests have not changed the conseling a great deal, but even there the national tests have more prestige and their results are more willingly accepted. When we used to say, "His arithmetic teacher feels that he will not be able to succeed in algebra," we would hear, "She never liked him anyway." But when we explain that this is not a test in algebra or geometry, but in the kinds of things he must be able to do in order to learn algebra and geometry, and that pupils all over the country who have made low scores have either failed in them or had great difficulty with them, that sounds reasonable and causes less argument.

Do achievement grades in algebra and geometry fall in line with the prediction of the aptitude test scores? In most cases, yes. In so large a percent of the cases that the teachers emphatically underwrite the tests. No statistical measurement is infallible. Pupils with high PLR's or IQ's usually do the best work in all subjects, yet you know that not all high IQ's do well. Sometimes they are lazy, or in poor health, or work too much outside of school, or have unnerving emotional instability at home, or are poor readers. Yet the mental test ratings are valuable measurements to have. So are the aptitude test scores.

At one school, in a group of 72 pupils taking elementary algebra, one pupil enrolled was in the *no* group. He made a grade of D. 85% of the A and B grades were made by pupils in the *surely* group. In another school, in a group of 91 pupils taking geometry, 73% of the A and B grades were in the *surely* group; 22% were in the maybe group; only one pupil with a low test score made a good grade. In this study, explanations were carefully given for every pupil whose achievement did not bear out the test prediction. They were absence from fourteen days up, poor study habits, lack of interest, and so on. In my own school one term, I followed carefully eleven pupils whose parents insisted on geometry against our advice, asking their teachers for frequent reports. Of these, seven failed, and four made only D.

What criticisms do the teachers make? Some teachers would prefer to follow the aptitude test predictions absolutely, without listening to parents' demands. In public schools we cannot be that arbitrary.

Some teachers feel that the arithmetic content of the algebra aptitude test does not weigh enough, since a pupil seriously deficient in arithmetic is impossible in algebra. This is a thoughtful criticism, but sufficient consideration of a pupil's grades in arithmetic would help here.

Several think that 145 in geometry and 37 in algebra as lower limits are too low and recommend that they be raised.

Some say that the PLR should be dropped from the geometry index as it was from the algebra index, that the test score alone is a better guide.

A few criticisms are made of poor co-operation between the schools giving the tests and the schools teaching the subjects. In a large school system like ours, there are bound to be occasional hitches in routine. In my own case, I could not ask for better cooperation. I am sure that any bad places could be easily ironed out with the help of the director.

Several schools speak of the problem of receiving pupils from parochial schools or out-of-town schools who have not had the aptitude tests. That problem can be dealt with, too, It was my worst headache at first, for there are a number of eight-grade parochial schools in our district, many of whose pupils come to us in the ninth grade, practically all electing algebra, but having had no tests. The first day of the term, after classes are over, I give them the aptitude test and assign them to classes or not according to the results. Pupils from out-of-town are given the test right away also. This testing is an added worry in the hectic first days of the semester, but the other teachers in the department help, and in the long run it is a much smaller worry than dealing with failing pupils who should never have taken the subject.

Teachers in other departments are co-operating now. Some of them even think that a good algebra or geometry aptitude test score is a better general measure than the PLR, and it is a long time since I have been accused of picking only good pupils for mathematics to keep down the per cent of failures.

Has the per cent of failures materially decreased since the aptitude tests have been used? Every teacher knows the answer to that one. No. Whether there is an ironclad rule that no more than a certain per cent may fail, or whether it is just understood that too many failures will make trouble, if a class is crowded with incompetents so that 40% or 50% should fail, you know what happens. Enough of those who are not the worst are passed to keep the per cent reasonable.

The Bureau of Statistics gave me the figures for failures in the city in algebra and geometry for the last ten years, both before and after the tests have been given. There is not much difference in percents, but the consensus of opinion of those who wrote to me is that a larger per cent of the 10,000 who passed 9B or 9A Algebra last year had a right to pass and really understood algebra than of those who took it ten years ago, and similarly, a larger per cent of the 6,000 who passed 10B or 10A Geometry last year are in the same class, and that therefore the teaching of the two subjects is definitely improved.

It is also true that many individual pupils have fared better because they have been guided into subjects that suit their abilities. For each individual to have that which is best for him, is the whole basis of democracy.

Notice to Subscribers

MEMBERS whose subscriptions expire in May 1946 will assist us greatly by renewing their subscriptions now.—Editor

Record of Attendance, Twenty-Fourth Annual Meeting, Cleveland, Ohio, February 22-23, 1946

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Florida	1
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Illinois	14
Iowa	4
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Michigan	7
Minnesota	1
Missouri	2
New Jersey	1
New York	13
North Carolina	1
Ohio	218
(Cleveland	99)
(Other Cities	119)
Oklahoma	1
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Wisconsin	3
Canada	1
TOTAL	299

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No. 11	Mathematics: A Study Guide for Teachers	19 pp.	25¢
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17	An Index to Visual and Auditory Aids and Materials	36 pp.	35¢
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Institute for Teachers of Mathematics

(ARITHMETIC—INTO—CALCULUS)

DUKE UNIVERSITY, AUGUST 8–17
W. W. RANKIN, *Director*
VERYL SCHULT, *Assistant Director*

PROGRAM (tentative)

Thursday, August 8, 1946

	Lectures	*Speakers*
8:00 P.M.	Atomic Energy	W. M. Nielsen, Duke
	Organization of Study Groups	Veryl Schult, Washington, D. C.

Friday, August 9, 1946

10:00 A.M.	Mathematics Laboratory	W. W. Rankin, Duke
8:00 P.M.	From Hypothesis to Conclusion in Science and Engineering	J. W. Cell, N. C. State
	Building Empirical Formulas Useful in Automotive Design	J. O. Allmen, Gen. Motors Research Laboratory

Saturday, August 10, 1946

10:00 A.M.	Mathematical Research on an Automobile Differential	J. O. Allmen
7:30 P.M.	Dinner	
	Naval Architecture	Admiral E. L. Cochrane, Navy Department, Bureau of Ships
9:30 P.M.	Social Hour	

Sunday, August 11, 1946

5:00 P.M.	Tea at Rankin Home	

Monday, August 12, 1946

10:00 A.M.	Mathematics of Watch Making	Representative of Hamilton Watch Co.
8:00 P.M.	Problems in Keeping Time	Representative of Hamilton Watch Co.
	Movie (to be selected)	

Tuesday, August 13, 1946

10:00 A.M.	Statistics as a Career	Gertrude Cox, N. C. State
8:00 P.M.	Reception (University House)	
	Geometry of Crystals	W. Berry, Duke
	Movie (to be selected)	

Wednesday, August 14, 1946

10:00 A.M.	How to Make and Use Nomographs	J. M. Thomas, Duke
8:00 P.M.	Mathematics in Industry	Representative of DuPont Co.
	Movie (to be selected)	

Thursday, August 15, 1946

10:00 A.M.	Rehabilitation of Geometry	W. W. Rankin, Duke
	Aids in the Study of Geometry	Frances Burns, Oneida, N. Y.
8:00 P.M.	The Mathematics of Radar	Representative of Sperry Gyroscope Corp.
	Movie (to be selected)	

Friday, August 16, 1946

10:00 A.M.	Mathematics and the Scientific Method	J. W. Cell, N. C. State
8:00 P.M.	Mathematics and Electronics	Representative of Sperry Gyroscope Corp.
	Party at Rankin Home (Watermelon)	

Saturday, August 17, 1946

10:00 A.M.	$X^2+Y^2+Z^2-R^2$	Veryl Schult
	Study Group Reports	
12:00 M.	Adjournment of Institute	

Study Groups

No. I. Enrichment of Mathematics

Leader—Miss Veryl Schult, Head of Mathematics Department, Washington, D. C. City Schools

This group will study the materials of the Mathematics Laboratory; books, instruments, models, charts, historical materials, curriculum studies, model and instrument making, mathematical tables, how to use such materials in the study of mathematics.

8:30–10:00 Daily

No. II. Consumer Mathematics

Leader—Miss Ruth Kimball, Washington, D. C. City Schools

This group will study the actual uses of mathematics in the work-a-day world, how to collect, and organize data of current experiences into some type of mathematical analysis (algebraic, graphical or statistical). The objective in this study will be to build a course of study suitable for high school students. Teachers are requested to bring with them special data for study in this group.

11:00–12:30 P.M. Daily

No. III. Measurement and Computation

Leader—Professor W. W. Rankin, Duke University

The work in this group will be in field work such as can be done in high school with: transit, level, sextant, planetable. This group will also study measuring instruments used in industry, trades, science, commerce, such as computing machines, slide rule, planimeter, computing scales, micrometer, scale drawing, etc.

2:30–4:00 P.M. Daily

No. IV. Applications of Mathematics to Science and Engineering

Leader—Professor J. W. Cell, North Carolina State College

Members of this group will get to see theoretical and practical mathematics join hands. The "spirit of discovery" will be encouraged. Emphasis will be placed on collecting a set of practical problems suitable for high school and college mathematics through calculus.

4:00–5:30 P.M. Daily

Teachers attending the Institute will be expected to join one or two of the study groups. They may visit other groups. Teachers are invited to bring to the study groups problems of special interest to them.

The work of the Institute will center around the Mathematics Laboratory.

EDITORIALS

The New President of the National Council of Teachers of Mathematics

DR. SHUSTER is a native of New Jersey, where his ancestors settled around 1680–1700. He graduated from high school at the age of 17 and spent the next three years teaching in New Jersey. During this time he originated what was later called the project method.

In February 1913 he graduated from the New Jersey State Normal School at Trenton and, after a half year of teaching at Dover High School, entered Columbia University where he secured the B.S., the A.M., and the Ph.D. degrees.

Dr. Shuster has had the following experience: Ungraded school 3 yr. Continuation school 5 yr., high school 12 yr., college 20 yr. During most of his life Dr. Shuster taught in two different kinds of schools at the same time. In 1929 he became head of the Mathematics Department of the New Jersey State Teachers College at Trenton. He has also taught in Bowling Green University and at Pennsylvania State College, and has taught at Teachers College, Columbia University, since 1926.

During World War I he served in the Navy as a Chief Petty Officer and after the war continued with the Federal Government as a senior cost accountant.

Dr. Shuster is author of a study of the Problems in Teaching the Slide Rule and co-author of Field Work in Mathematics and the Real Life series of arithmetics. He was a contributor to the Third Yearbook of the National Council, is on the editorial board of the National Mathematics Magazine, and has written numerous magazine articles. Dr. Shuster joined the National Council in 1921, was chairman of the Defense Committee of the National Council for 1941, and a Director 1944–1947. He is a member of the National Council of Teachers of Mathematics, the American Mathematical Society, the Mathematical Association of America, the American Association for the Advancement of Science. He is also a member of Phi Delta Kappa honorary educational fraternity, the Torch Club International and numerous local mathematics and other associations. He has also served on several national and state committees interested in improving mathematical education.

THE MATHEMATICS TEACHER wishes to congratulate both Professor Shuster and the Council on his election to the presidency, and to wish him the greatest success.—W.D.R.

This issue contains all of the papers that were read at the recent annual meeting of the National Council of Teachers of Mathematics at Cleveland. Naturally one does not get the full report of the meeting since some of the speeches were not written down. This was particularly true of the report of Dr. Schorling's Commission on Post War Plans. This is one distinct advantage of being present at such meetings. And this meeting was an unusually good one. Every teacher of mathematics who possibly can should plan to attend at least a few of these meetings. They do something for a teacher that a written report as good as it may be cannot convey. Let us all try to attend the next meeting if it is possible to do so. Meetings of this kind do much for our national spirit and morale.

Great credit for the success of the Cleveland meeting should go to the mathematics teachers there who supported the programs magnificently. In particular Mr. and Mrs. A. Brown Miller and other leaders among the mathematics teachers in Cleveland are to be congratulated for the fine work they did. They always give the Council excellent support.—W.D.R.

◆ IN OTHER PERIODICALS ◆

By NATHAN LAZAR

Midwood High School, Brooklyn 10, New York

1. Brownell, W. A., "Arithmetic Learnings," *Instructor*, 54: 14+, June 1945.
2. Brownell, W. A., "Meaning Theory in Arithmetic," *Instructor*, 54: 31+, September 1945.
3. Brownell, W. A., "Teaching of Mathematics in Grades I through VI," *Review of Educational Research*, 15: 276–288, October 1945.
4. Brueckner, L. J., "Vitalizing Arithmetic Instruction," *National Elementary Principal*, 25: 24–28, September 1945.
5. Burr, H., "Mathematics in General Education," *California Journal of Secondary Education*, 20: 394–398, November 1945.
6. Buswell, C. T., "Selected References on Elementary School Instruction; Arithmetic." *Elementary School Journal*, 46: 160–162, November 1945.
7. Cattell, R. B., "Personality Traits Associated with Abilities; With Verbal and Mathematical Abilities," *Journal of Educational Psychology*, 36: 475–486, November 1945.
8. Deans, E., and Sanders, L. P., "Head Start in Arithmetic," *Parents Magazine*, 20: 22+, October 1945.
9. Eagle, E., and Palmer, G. W., "Reactions to the National Council Second Report," *California Journal of Secondary Education*, 20: 373–376, November 1945.
10. "Efforts Towards the Improvement of Mathematics Teaching—A Symposium," *California Journal of Secondary Education*, 20: 369–399, November 1945.
11. Garrett, A. B., and Fawcett, H. P., "Our Students Do Not Know Arithmetic," *Ohio Schools*, 23: 200–201+, May 1945.
12. Guiler, W. S., "Difficulties Encountered by College Freshmen in Fractions," *Journal of Educational Research*, 39: 102–115, October 1945.
13. Guiler, W. S., "Difficulties in Fractions Encountered by Ninth-Grade Pupils," *Elementary Schools Journal*, 46: 146–156, November 1945.
14. Gunderson, A. G., "Problem Solving in the Primary Grades," *Instructor*, 54: 36–37+, October 1945.
15. Hall, J. V., "Oral Aids in Problem Analysis," *Instructor*, 55: 24+, January 1946.
16. Halliday, F. F., "Two and Two Make Four," *School* (*Elementary Edition*), 34: 41–44, September 1945.
17. Hartung, M. L., "Teaching of Mathematics in High School and Junior College," *Review of Educational Research*, 15: 310–320, October 1945.
18. Heely, R. B., "Using Seventh and Eighth Grade Mathematics as a Pattern of Creative Ability," *Education*, 66: 183–189, November 1945.
19. Kinney, L. B., and Freeman, F. N., "Manner in Which Pupils Learn Mathematics," *California Journal of Secondary Education*, 20: 381–387, November 1945.
20. Lyda, W. J., and Franzén, C. G. F., "Study of Grade Placement of Socially Significant Arithmetic Problems in the High School Curriculum; Abstract of Doctor's Dissertation," *Journal of Educational Research*, 39: 292–295, December 1945.
21. McIntyre, J. V., "Filling in the Gaps," *School* (*Elementary Edition*), 34: 411–413, January 1946.
22. Meder, E. M., "Teacher Education in the Natural Sciences and Mathematics," *Review of Educational Research*, 15: 321–330, October 1945.
23. Moore, C. M., "Houses That Arithmetic Built," *Instructor*, 54: 41+, September 1945.
24. Moser, H. E., "Teaching of Mathematics in Grades VII and VIII," *Review of Educational Research*, 15: 298–300, October 1945.
25. "New Approach to School Mathematics," *Nature*, 156: 330, September 15, 1945.
26. Payne, O. R., "Arithmetic Troubles," *Parents Magazine*, 20: 124–125, November 1945.
27. Perell, J., "Success in Teaching Mathematics," *School* (*Secondary Edition*), 34: 238–241, November 1945.
28. Ralph, M., "When Shall We Begin Arithmetic?" *Sierra Education News*, 41: 20, November 1945.
29. Riddel, R. A., "Remedial Work in Arithmetic," *School* (*Elementary Edition*), 34: 157–159, October 1945.
30. "Roman Numbers," *Instructor*, 54: 33–34, September 1945.
31. Rosenberg, R. R., "State and Federal Civil Service Arithmetic Examination Problems," *Business Education World*, 26: 199–200, December 1945.
32. Rutan, E. J., "Teaching the Language of Mathematics," *Nation's Schools*, 36: 43–44, July 1945.
33. "San Diego Schools Arithmetic Methods," *American School Board Journal*, 111: 30, December 1945.
34. Sherer, L. M., "Mathematics in Elementary Schools," *California Journal of Secondary Education*, 20: 388–393, November 1945.
35. Siemens, C. H. "Some Problems in Secondary Mathematics," *California Journal of Secondary Education*, 20: 377–380, November 1945.
36. Smitheran, F. B., "Is Rough Work Necessary?" *School* (*Elementary Edition*), 34: 233–236, November 1945.
37. Strangways, D. H., "Analysis and Proof of a Deduction in Geometry," *School* (*Secondary Edition*), 34: 234–237, November 1945.
38. Whitney, C., "Meaningful Way," *Journal of Education*, 128: 204–206, September 1945.
39. Wynne, M. T., "Teaching Helps in Number and Music," *Grade Teacher*, 63: 30+, December 1945.
40. Zant, J. H., "United States Armed Forces Institute Examinations in Mathematics as a Means of Determining Credit in Engineering Mathematics," *Journal of Engineering Education*, 36: 170–174, November 1945.

NEWS NOTES

The twenty-third Annual joint meeting of the Louisiana-Mississippi Section of the Mathematical Association of America and the Louisiana-Mississippi Branch of the National Council of Teachers of Mathematics was held at the Louisiana Polytechnic Institute at Ruston, Louisiana, on Friday and Saturday, March 22 and 23, 1946. The programs follows:

THE LOUISIANA-MISSISSIPPI SECTION OF THE MATHEMATICAL ASSOCIATION OF AMERICA

Room 113, Bogard Hall—2:30 P.M. Friday, March 22

F. C. Gentry, Louisiana Polytechnic Institute Secretary

Election of a Chairman, Announcements and Appointment of Committees

1. Esthetic and Moral Implications in the Arc of the Covenant—B. E. Mitchell, Millsaps College.
2. Smoothing Effects of Linear Oscillators—W. L. Duren, Tulane University.
3. The Future Importance of Secondary Mathematics in the Curriculum is now in the hands of the College Teachers—Houston T. Karnes, Louisiana State University.
4. The Presentation of Certain Application Problems in the Calculus—P. K. Smith, Louisiana Polytechnic Institute.
5. Some Topics in Applied Mathematics—C. D. Smith, Mississippi State College.

Joint Banquet

L. P. I. Dining Hall, 7:00 P.M., Friday, March 22

Welcome—President Claybrook Cottingham, Louisiana Polytechnic Institute

James Beckett, Central High School, Jackson, Miss.

Response—B. E. Mitchell, Millsaps College.

Address: The Motionless Arrow—Prof. Nathan Altschiller-Court, University of Oklahoma.

LOUISIANA-MISSISSIPPI BRANCH OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Room 133, Bogard Hall, 9:00 A.M. Saturday, March 23

Miss Daisy White, Vice-Chairman, presiding
Mr. James Beckett, Secretary

1. A Rythmic Approach to Geometry—Margaret Rae Davis, Southeastern Louisiana College.
2. The Job of the Mathematics Teacher—O. J. Thayer, Louisiana State University Laboratory School.
3. The Work of the National Council of Teachers of Mathematics—Nathan Altschiller-Court, University of Oklahoma.
4. Current Problems in Secondary Mathematics (Round Table)—P. K. Smith, Louisiana Polytechnic Institute.
5. Business Meeting and Election of Officers.

LOUISIANA-MISSISSIPPI SECTION OF THE MATHEMATICAL ASSOCIATION OF AMERICA

Room 113, Bogard Hall—10:30 A.M., Saturday, March 23

Address: On a Pencil of Ruled Quartics—Nathan Altschiller-Court, University of Oklahoma.

Business Meeting and Election of Officers.

OFFICERS 1942–46

LOUISIANA-MISSISSIPPI BRANCH OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

W. H. Bradford, John McNeese, Junior College—Chairman.
Miss Daisy White, Ruston High School—Vice-Chairman.
James Beckett, Central High School, Jackson—Secretary.
Houston T. Karnes, Louisiana State University—Recorder.

LOUISIANA-MISSISSIPPI SECTION OF THE MATHEMATICAL ASSOCIATION OF AMERICA

James A. Ward, Delta State Teachers College—Chairman (resigned).
B. O. Van Hook, Millsaps College—Vice-Chairman (resigned).
S. T. Santers, Jr., Southwestern Louisiana Institute (deceased)—Vice-Chairman.
F. C. Gentry, Louisiana Polytechnic Institute—Secretary.

According to the New York *Times* of March 27, 1946, Professor Fletcher Durell, for many years one of New Jersey's best known educators and father of Thomas B. Durell, assistant State Commissioner of Education in charge of elementary schools, died in his home in this Cape May County community last night at the age of 86. He had been ill since January, when he fractured a leg in a fall.

Born in Clarksburg, N. J., Professor Durell was graduated from Princeton University in 1879, in the same class as Woodrow Wilson, and until 1881 did graduate work in mathematics on the John Kennedy Scholarship at the University of Leipzig in Germany.

He taught mathematics at Pennington Seminary from 1881 to 1883, and was Professor of Mathematics and Astronomy at Dickinson College for eleven years.

Professor Durell was head of the department of mathematics at the Lawrenceville School from 1895 until he retired in 1925. He had been a member of the American Mathematical Society, the Mathematical Association of America, the American Association of Science and the Princeton Club of Philadelphia. He also had served as president of the board of managers of the State colony for feeble-minded at Woodbine.

He was the author of a number of books, among them *A New Life in Education* in 1914; *Cooperation—Its Essence and Background*, 1936, and *Mathematical Adventures*, 1938.

Professor Durell was nationally known for his textbooks, which included a series of algebras, another of geometries and a volume on trigonometry, all of which have been used by high schools throughout the country, and a series of arithmetics for elementary schools.

Last July Professor and Mrs. Durell celebrated their sixtieth wedding anniversary. Surviving besides Mrs. Durell are a son; a daughter, Miss Marian Durell; four grandchildren, Mrs. Palmer M. Way of Wildwood Crest, N. J., and John, Henry and Ann Durell, all of Princeton, and two great grand children.

THE MATHEMATICS TEACHER was recently given a citation by Mr. Fred M. Vinson, secretary of the treasury of the United States for distinguished services rendered in behalf of the War Finance Program.

Unified in authorship, methods, and objectives here are four books to make mathematics interesting and understandable for your students.

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